NEW ORDINARY LEVEL MATHEMATICS RESEARCH BOOK

(DETAILED)

SENIOR ONE TO SENIOR FOUR

"LEARNER'S RESEARCH BOOK"

BASED ON THE NEW LOWER SECONDARY CURRICULUM

By

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Preface

This learner's research book has been written in line with the revised mathematics syllabus for the new lower secondary curriculum.

The main reason as to why We have written this book, is to make research easier to learners as they are making their own notes in mathematics. Therefore, this is a detailed research book for the new revised mathematics ordinary level syllabus.

Also this book has been equipped with a number of trial questions per topic such that learners master mathematics concepts as they work out those questions since practice makes mathematics easier.

This learner's research book is one of the materials which are to be used to support the teaching and learning process of the new lower secondary curriculum .

Lwanga Books Ltd feels confident that this Book will be of immense value to both the learners and the teachers.

Any suggestions for improvement of this book are most welcomed, thanks.

"It is not what We do for you but what We will teach you to do for and by yourselves that will eventually make you successful beings in the society"

Acknowledgement

Lwanga Books Limited is deeply indebted to all those who participated in the development of **Lwanga William S1-S4 Mathematics Learner's Research Book.**

Special thanks go to **Mr. Lwanga William**, the CEO Lwanga Books Ltd for his valuable insights and advice on all publishing matters.

We would like to express our sincere appreciation to all those who worked tirelessly towards the production of this learner's research book.

First and foremost, we would like to thank our families and friends for supporting all our initiatives both financially and spiritually, Lwanga William's parents; **Mr. William Lwanga** and **Mrs. Harriet Lwanga,** his brother; Mr. Nsubuga Grace.

The initiative and guidance of the publishing partners, Ministry of Education and Sports (MoES) and National Curriculum Development Centre (NCDC) in development and implementation of the New Lower Secondary Curriculum are highly appreciated.

We thank God for the wisdom He has given us to produce this volume of work. May the Almighty God bless all the students that will use this book with knowledge of making their own notes as they are making research...….**AMEN**.

We welcome any suggestions for improvement to continue making our service delivery better.

NB: "Search" {lwanga william} on youtube and subscribe (also tap on the notification bell) to that you-tube channel and watch the subject based project lessons that are on-going. " subscription is for free"

NUMBER BASES

Definition: Number bases are different ways of using the same number. We use a system called base 10, or decimal, for our arithmetic, but there are as many number bases as there are numbers.

These common bases also have proper names, shown inparentheses:

- base 2 (binary)
- base 8 (octal)
- base 10 (decimal)
- base 12 (duodecimal)
- base 16 (hexadecimal)

We are accustomed to writing numbers in base ten, using the symbols for 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. For example, 75 means 7 tens and five ones. However numbers can be written in any number base.

If we use base 8 instead of base ten, then 75 is written as 113 whichdenotes one sixty four (8^2), one eight (8^1) and 3 units (instead of hundreds, tens and units).

Base 2 is particularly useful as it only requires two symbols, for zero and one, and it is the way numbers are represented in computers.

Just as, in base ten, the columns represent powers of 10 and have'place value' 1, 10, 10^2 , 10^3 etc. (reading from right to left), so in base 2, the columns represent powers of 2.
Hence the number 1001011 denotes (reading from right to left):
1 unit (2^0), 1 two (2^1), no fours (2^2), 1 eight (2^3), no six teens (2^4), no thirty twos (2^5), 1 sixty four (2^6).

The number 1001011 in base 2 is the same as the number 75 inbase ten.

As another example, we use the symbols 0, 1, 2, 3 and 4 to represent numbers in base 5. The columns in base 5 have 'place value' 1, 5, 25, 125, 625 etc reading from right to left. The number 75 in base ten is the same as the number 300 in base five, that is 3 twenty fives, no fives and no units.
The number 4102 in base 5 denotes 2 units, no fives, 1 twenty five and 4 one hundred and twenty fives making altogether 527 in base ten.

Writing the number 75 in base six we get 203, which represents 2 thirty sixes, no sixes and 3 units.

We have seen that 75 (base10), 1001011 (base 2), 300 (base 5),113 (base 8), and 203 (base 6) all represent the same number.

Similarly, we can write 75 in any base we choose, and we can writeall numbers in any base.

To write numbers between 0 and 1, we use negative powers of the base. For example, in base 2 we use halves, quarters, eighths, sixteenths etc instead of the tenths, hundredths, thousandths etc. which we use in base ten.

So if we write 11.11 in base 2 this denotes $2^1 + 2^0 + 2^{-1} + 2^{-2}$. The equivalent in base 10 is $2 + 1 + {}^1/_2 + {}^1/_4$, that is, 3.75 in base 10.

Converting from Decimal to any other base

Example

Express 5213_{ten} with a base 8.

To express 5213_{ten} with a base 8 we divide repeatedly by 8, and the remainders in reverse order give the required number.

8	5213	
8	651	rem 5
8	81	rem 3
8	10	rem 1
8	1	rem 2
	0	rem 1

$\therefore 5213_{ten} = 12135_{eight}$

Example

Express 21_{ten} with a base 2.

2	21	
2	10	rem1
2	5	rem0
2	2	rem1
2	1	rem0
	0	rem1

$\therefore 21_{ten} = 10101_{two}$

Converting to decimal from any other base.

One method is to write out the number in full:

$432_{five} = (4 \times 5^2) + (3 \times 5) + (2 \times 1)$

$= 100 + 15 + 2 = 117_{ten}$

$10101_{two} = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2)$

$+ (0 \times 2) + (1 \times 1)$

$= 16 + 0 + 4$

$+ 0 + 1 = 21_{ten.}$

$526_{eight} = (5 \times 8^2) + (2 \times 8) + (6 \times 1) = 320 + 16 + 6 = 342_{ten.}$

A quicker method can often be done mentally. Thus, for 526_{eight}, we say:
5 x 8 is 40, plus 2 makes 42; 42 x 8 is 336, plus 6 makes 342.

$\therefore 526_{eight} = 342_{ten}$.

Conversion of others

Whenever we want to change one base to another base we mustconvert it to base ten first.

For example,

(i) Change $8et1_{twelve}$ to base 7.

(ii) Find n if $100001_{two} = 45_{n}$.

Solutions

(i) Changing 8et1 $_{twelve}$ to base ten,

$8et1_{twelve} = (8 \times 12^3) + (11 \times 12^2)$

$+ (10 \times 12^1) + (1 \times 12^0)$

$= 13824 + 1584 + 120 + 1$

$= 15529$

Converting 15529 to base seven we have

7	15529
7	2218 rem 3
7	316 rem6
7	45 rem 1
7	6 rem 3

Hence, $8et1_{twelve} = 63163_{seven}$

(ii) Converting 100001 two to base ten we have

$(1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (0 \times 2^2)$

$+ (0 \times 2^1) + (1 \times 2^0)$

$1 \times 32 + 0 \times 16 + 0 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1$

$= 32 + 0 + 0 + 0 + 0 + 1 = 33$

Converting 45 $_n$ to base ten we have

$(4 \times n^1) + (5 \times n^0) = 4n + 5$

$\therefore 33 = 4n + 5$

$4n = 33 - 5 = 28$

$n = 7$

Calculations with any base other than 10

When performing the ordinary operations of arithmetic in any base other than ten, we must remember that the digits no longer denote successive powers of ten, but of the base which is being used. Thismeans that the carrying figures are found by dividing by the base, not by ten.

Example*

Add 232_{five} to 344_{five}.

We say 2 + 4 = six = (1x5) + 1. Put down 1

And carry 1; 3+4+1 = eight = (1x5)+3;

Put down 3 and carry 1; 2+3+1 = six = (1x5) +1; put down 1 and carry 1 into the fourth place from the right.

$$\begin{array}{r} 232 \\ +344 \\ \hline 1131 \end{array}$$

Ans. 1131_{five}

Example

Subtract 43_{eight} from 72_{eight}.

We cannot subtract 3 from 2, so we borrow one from the eights column and call it eight in the ones column, making 2+8, or ten; 3 from 10 leaves 7. In the eights column we can either add 1 to 4 and subtract 5 from 7, or take 1 from the 7, leaving 6, and then subtract 4 from 6. In either case it leaves 2.

$$\begin{array}{r} 72 \\ -43 \\ \hline 27 \end{array}$$

Ans. 27_{eight}

Example

Multiply 6734_{eight} by 27_{eight}.

When multiplying by the 2, we say 2x4 = eight

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= (1x8) + 0; put 0 and carry 1; (2x3) +1= seven, so
we put seven; 2x7 = fourteen = (1x8) + 6, so we put
6 and carry 1; (2x6) +1 =thirteen = (1x8) +5, so we put 5 and carry 1.
When multiplying by the 7 we say,
7x4 = 28.
= (3x8) +4; put 4 and carry 3; (7x3) +3 = twenty four
= (3x8) + 0; put 0 and carry 3; (7x7)+3 = fifty two = (6x8) +4; put 4 and carry 6; (7x6) + 6 = forty eight = (6x8) +0; put 0 and carry 6. The addition is then done as in example *.

$$\begin{array}{r} 6734 \\ \times \quad 27 \\ \hline 15670 \\ +\ 60404 \\ \hline 237304 \\ \hline \end{array}$$

Ans. 237304$_{eight}$

Example

Divide 110111_{two} by 101_{two}

Solution

```
101]110111[1011
   101
     111
     101
      101
      ...
```

Ans. 1011_{two}

NOTE: To solve problems with number bases, it is easiest to write everything in base ten and then finally express the answer in the required base. However, you can use whichever method you prefer.

EXERCISE

1. Convert to base ten: 43_{eight}, 2314_{eight}, 7_{eight}, 27_{eight}, 127_{eight}.

2. Express with base 8: 47_{ten}, 252_{ten}, 725_{ten}, 866_{ten}, 9_{ten}, 39_{ten}.

3. Write 5432_{ten} with base: (i) five (ii) three (iii) six.

In numbers 4 to 15, carry out the calculations and give the answers in the base indicated.

4. $2102_{three} + 2202_{three}$
5. $11011_{two} + 1011_{two}$
6. $72_{eight} + 41_{eight} + 267_{eight}$
7. $201_{three} - 122_{three}$
8. $11011_{two} - 1101_{two}$
9. $152_{eight} \times 43_{eight}$
10. $124_{five} \times 32_{five}$
11. $11111_{two} \times 1001_{two}$
12. $11001_{two} \times 111_{two}$
13. $3256_{eight} \div 46_{eight}$
14. $3212_{five} \div 14_{five}$
15. $10010_{two} \div 110_{two}$

Point notation

Example

Express 0.75_{ten} in binary.

Solution:

In binary, place values after the point are as shown below:

$$0.\ \overline{\left(\frac{1}{2}\right)_{ten}}\ \overline{\left(\frac{1}{4}\right)_{ten}}\ \overline{\left(\frac{1}{8}\right)_{ten}}\ \overline{\left(\frac{1}{16}\right)_{ten}}\ \text{etc}$$

$$\text{But } 0.75_{ten} = \left(\frac{3}{4}\right)_{ten} = \left(\frac{1}{2}+\frac{1}{4}\right)_{ten}$$

Therefore, $0.75_{ten} = 0.11_{two}$.

Example

Express 0.12_{six} as a fraction in base ten.

Solution:

$$0.12_{six} = \left(1\times 6^{-1} + 2\times 6^{-2}\right)_{ten} = \left(\frac{8}{36}\right)_{ten} = \left(\frac{2}{9}\right)_{ten}$$

Example

Work out:
(i) $28.57_{nine} + 6.34_{nine}$
(ii) $30.241_{five} - 14.143_{five}$.

Solution:

$$\text{(i)}\ \begin{array}{r} 28.57_{nine} \\ +\ \ 6.34_{nine} \\ \hline 36.02_{nine} \\ \hline \end{array} \qquad \text{(ii)}\ \begin{array}{r} 30.241_{five} \\ -14.143_{five} \\ \hline 11.043_{five} \\ \hline \end{array}$$

EXERCISE

1. Express 1001.01_{two} in base ten in point notation.

2.Using point notations express 6.41_{eight} in base four.

3. Express 0.6_{eight} in base ten in the form $\frac{a}{b}$.

4. Use point notation to express 45.3_{six} in base ten.

5. Express $\left(\frac{2}{10}\right)_{ten}$ in base ten.

6. Express $\left(\frac{3}{8}\right)_{ten}$ in binary in point notation.

7.Use point notation to express $\left(\frac{5}{12}\right)_{ten}$ in base eight

Work out the following:

8. $0.111_{two} + 0.011_{two}$

9. $11.1_{two} - 0.111_{two}$

10. $2.123_{four} + 0.313_{four}$

11. Find n: (i) $45_n = 41_{ten.}$

(ii) $103_n + 26_n = 131_n$

Remember; A number base is the number of digits or combination of digits that a system of counting uses to represent numbers. A base can be any whole number greater than 0. The most commonly used number system is the decimal system, commonly known as base 10. In everyday life, we countor estimate quantities using groups of ten items or units. This may be so because, naturally, we have ten fingers. For example, when we count ten, i.e. we write 10 meaning one group of 10 and no units. A quantity like twenty five, written as 25 means 2 groups of 10 and 5 units Suppose instead we had say 6 fingers.

1. How, in your opinion would we do our counting?

2. If we had eight fingers, how would we count?

NOTE

1 The digits of a number in any base are less than the base itself

2 The digits 10 and 11 are represented by t and e respectively in number bases

3 For digits above 11 are represented by alphabetic letters of your choice

4 The names of some number systems is as given below

NUMBER SYSTEM	NAME	BASE VALUE
Base three	trinary base	3
Base four	quaternary base	4
Base five	quinary base	5
Base six	seximal base	6
Base seven	septimal base	7
Base eight	octal base	8
Base nine	nonary base	9
Base ten	decimal base	10
Base twelve	duodecimal base	12
Base sixteen	hexadecimal base	16

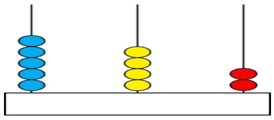
Bases are used in day today life.Therefore copy and complete the table below by giving some reallife situations were bases are used

Real life situation	Base	Reason for the base chosen
Days of the week	Base Seven	Seven days in a week
Football team	Base eleven	There are 11 players per team
Months of a year	Base twelve	Twelve months in a year

Identifying numbers of different bases on an abacus

Which possible base does each abacus below represent.

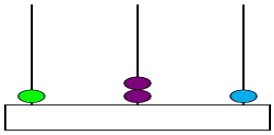
Hundreds Tens Ones



(a)

Three

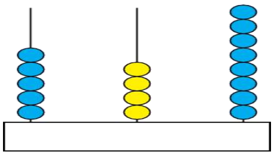
threes Threes Ones



(b)

Nine

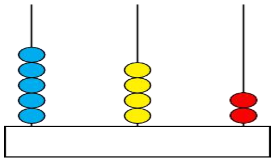
nines Nines Ones



(c)

❖ Write down the numbers represented on the abaci below.

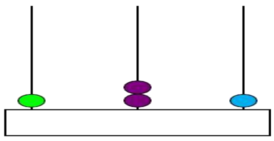
Hundreds Tens Ones



(a)

Three

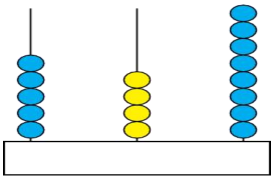
threes Threes Ones



(b)

Nine

nines Nines Ones

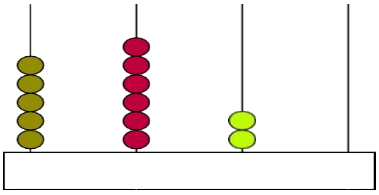


(c)

Seven

Seven Seven

Sevens sevens Sevens Ones



(d)

✧ Numerals are digits(or symbols) that are used for writing numbers in a given base.The digits arealways less than the base itself. study the table below and ll in the gaps.

NUMBER SYSTEM	BASE VALUE	NUMERALS	EXAMPLE
Base 2	2	0,1	1111_{two}
Base 3	3		
Base 4	4		
Base 5		0,1,2,3,4	
Base 6			
Base 7			
Base 8			457_{eight}
Base 9			
Base 10			
Base 12	12	0,1,2,3,4,5,6,7, 8,9,t,e	$te5_{twelve}$
Base 16			

Place Values Using the Abacus

The representation of numbers on an abacus helps in identifying the place value of digits in any base.

1. Make abaci for the following number bases.

(a) 220_{four}

(b) 7562_{eight}

(c) 654_{nine}

(d) 5974_{eleven}

2. State the place value and value of digit for each numeral in the following numbers:

(a) 1432_{five} (c) 3412_{six} (e) 6542_{eight} (g) 45.62_{eight}

(b) 111_{two} (d) 431_{seven} (f) 4.1234_{ten} (h) 456.212_{ten}

Converting Numbers

Numbers can be converted from one base to another, and when you do this, you get the samenumbers written in different bases.

Converting from any base to base ten

EXAMPLES

Convert the following to base ten

1. 222_{four}

$222_{four} = (2 \times 4^2) + (2 \times 4^1) + (2 \times 4^0)$

$= (2 \times 4 \times 4) + (2 \times 4) + (2 \times 1)$

$= 32 + 8 + 2$

$= 42_{ten}$

2. $ee0_{twelve}$

$ee0_{twelve} = (e \times 12^2) + (e \times 12^1) + (0 \times 12^0)$

$= (11 \times 12 \times 12) + (11 \times 12)$

$+ (0 \times 1)$

$= 1584 + 132 + 0$

$= 1716_{ten}$

1. Convert 68.3_{nine} to base ten

2. Convert the following binary numbers to base 10:

(a)	110	(d)	1101	(g)	1111111
(b)	1111	(e)	10001	(h)	11001101
(c)	1001	(f)	11011	(i)	111000111

3. A particular binary number has 3 digits.

(a) What are the largest and smallest possible binary numbers?

(b) Convert these numbers to base 10.

Converting from base ten to other bases

- We use BNR
- Divide the number repeatedly by the required bases
- The remainder in reverse order gives the required number

1.Convert 19_{ten} to base two

B	N	R
2	19	1
2	9	1
2	4	0
2	2	0
	1	

$19_{ten} = 10011_{two}$

2.Convert $85_{ten}$to base eight

B	N	R
8	85	5
8	10	2
	1	

$85 = 125_{eight}$

3.Convert 762_{eight} to base seven

$762_{eight} = (7 \times 8^2) + (6 \times 8^1) + (2 \times 8^0)$

$= (7 \times 8 \times 8) + (6 \times 8) + (2 \times 1)$

$=448 + 48 + 2$

$= 498_{ten}$

B	N	R
7	498	1
7	71	1
7	10	3
	1	

$762_{eight} = 1311_{seven}$

4. Convert 32_{five} to base two

$32_{five} = (3 \times 5^1) + (2 \times 5^0)$

$= (3 \times 5) + (2 \times 1)= 15 + 2= 17_{ten}$

B	N	R
2	17	1
2	8	0
2	4	0
2	2	0
	1	

$32_{five} = 10001_{two}$

5. Convert 5432_{six} to base twelve

$5432_{six} = (5 \times 6^3) + (4 \times 6^2) + (3 \times 6^1) + (2 \times 6^0)$

$= 1080 + 144 + 18 + 2 = 1244_{ten}$

B	N	R
12	1244	8
12	103	7
	8	

$5432_{six} = 878_{twelve}$

Operation on Numbers in Various Bases

Addition of bases

- If the sum of the digits exceeds the base, divide that sum by the base then write down theremainder and carry the whole number.

Subtraction of bases

- In case of borrowing the new value is the sum of the base and the digit which was small

1. Subtract 342_{eight} from 537_{eight}

$$\begin{array}{r} 5\,3\,7_{eight} \\ -\,3\,4\,2_{eight} \\ \hline 1\,7\,5_{eight} \end{array}$$

2. Subtract 432_{six} from 514_{six}

$$\begin{array}{r} 5\,1\,4_{six} \\ -\,4\,3\,2_{six} \\ \hline 4\,2_{six} \end{array}$$

Multiplication of bases

- Find the product of any two numbers as we do in base ten
- Divide this product by the base number
- Write the remainder and carry the quotient to the next place value position

Division of bases

- Convert each number base to base ten
- Divide the two numbers in base ten
- Convert the result back to the required base

EXAMPLES

1. Divide 1331_{four} by 121_{four}

First Convert 1331_{four} and 121_{four} to base ten and then finally express the answer in basefour.

Converting 1331_{four} to base ten

$1331_{four} = (1 \times 4^3) + (3 \times 4^2) + (3 \times 4^1) + (1 \times 4^0)$

$= 64 + 48 + 12 + 1 = 125_{ten}$

Converting 121_{four} to base ten

$121_{four} = (1 \times 4^2) + (2 \times 4^1) + (1 \times 4^0)$

$= 16 + 8 + 1 = 25_{ten}$

Dividing the numbers in base ten

$125 \div 25 = 5$

Converting to base four

B	N	R
4	5	1
	1	

$1331_{four} \div 121_{four} = 11_{four}$

More Exercise

1. Work out the following;

i) $3333_{five} + 1342_{six}$

ii) $312_{four} + 145_{six} - 10011_{two}$

iii) $22_{six} + 176_{eight}$ iv) $232_9 + 277_9$

2. Find base x give that ;

i) $304_x = 422_5 + 47_8$. ii) $25_x = 21$

iii) $102_x = 38_{ten}$ iv) $204_x = 242_{eight}$

v) $103_x + 26_x = 131_x$

vi) $45_x = 10001_{two}$ vii) $125_x = 85_{ten}$

viii) $45_x = 41_{ten}$

3. Given that $212_{\mathbf{n}} = 25_{\mathbf{nine}}$, find the base that **n** represents

Given that: i) $10111001_{five} = 271_n$,

ii) $212_n = 25_{nine}$ find the value of base **n**.

WORKING WITH INTEGERS

Numbers

Natural numbers (**counting numbers**).

These are numbers used in counting.

Thus, $\{Natural\ numbers\} = \{Counting\ numbers\}$
$= \{1,2,3,4,5,\ldots\}$

Whole numbers.

These are counting numbers including zero.
i.e. $\{Whole\ numbers\} = \{0,1,2,3,4,5\ldots\}$

Integers:

This is a set of whole numbers together with negative whole numbers. We can write:

$\{Integers\} = \{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

Factors and Multiples.

A whole number which divides exactly into another is said to be a factor of that number. Thus, if one number goes exactly into another number, the first number is called a **factor** of the second and the second number is called a **multiple** of the first.

Thus 3x4 = 12, ∴ 3 and 4 are factors of 12. Again 2x6 = 12, 2 and 6 are also factors of 12. Also 12 is a multiple of any of the numbers 2,3, 4, or 6, since each of these numbers goes exactly into 12.

A **prime** number is a number which has only two factors, itself and 1.Thus 2, 3, 5, 7, 11, 13, 17, 19, 23, …are all prime numbers.

A factor which is a prime number is called a **prime factor**.

Thus 3 is a prime factor of 12 and so is 2.

Note:

- ❖ Every number is a factor of itself.
- ❖ 1 has only itself as a factor and therefore it is not a prime number.
- ❖ Numbers that have more than two factors are called **composite numbers.**

Composite numbers can be expressed as products of their prime factors. We obtain prime factors of a number by successive division of the number starting with the least possibleprime factor.

Example

Express 36 in terms of its prime factors.

Solution:

2	36
2	18
3	9
3	3
	1

So $36 = 2\times2 \times3\times3 = 2^2\times3^2$

Example

Express 4312 in prime factors.(work it out)

Note: The above process of writing numbers in terms of their primefactors is called **prime decomposition** or **prime factorization.**

Exercise

1. List all the factors of each of the following numbers:
(a) 8 (b) 12 (c) 20
(d) 11 (e) 45 (f) 32

2. Express the following numbers as products of their prime factors.

(a) 8	(b) 42	(c) 90
(d) 240	(e) 72	(f) 1024
(g) 360	(h) 800	(i) 625
(j) 280	(k) 102	(l) 3465
(m) 1000	(n) 1764	(o) 3969
(p) 1728	(q) 9000	(r) 2744
(s) 9520	(t) 6936	(u) 1440

H.C.F and L.C.M

The **Highest Common Factor** (usually written as H.C.F) of two or more numbers is the greatest number which is a factor of each of them.

The **Lowest Common Multiple** (L.C.M) of two or more numbers is the least number which is a multiple of each of them.

To find the HCF of two or more numbers:

(i) Express each of the numbers as a product of prime factors,

(ii) Pick out the least power of each common factor. The product of these gives the HCF or GCF

To find the LCM we proceed as follows:

(i).If 2 divides any of the numbers, divide through by 2 as manytimes as possible. Write down, as it is, any number not divisible by 2.

(ii).Go to 3 and repeat the process. Then go to 5, 7, 11, … If anumber does not divide any of the numbers, skip it.

(iii).Proceed as in (ii) until you arrive at a row of 1s.

(iv).Pick the highest power of each of the prime factors that appear. The factors need not be common. Multiply them.

Example

Find the H.C.F and the L.C.M of 84, 1386 and 210.

Solution

2	84
2	42
3	21
7	7
	1

$84 = 2^2 \times 3 \times 7$

2	1386
3	693
7	231
7	77
11	11
	1

$1386 = 2 \times 3^2 \times 7 \times 11$

2	210
3	105
5	35
7	7
	1

$210 = 2 \times 3 \times 5 \times 7$

The common factors available are 2, 3 and 7.
The ones with leastpower are 2^1, 3^1 and 7^1.
Therefore, HCF = $2^1 \times 3^1 \times 7^1 = 42$.

The prime factors available are 2, 3, 5, 7 and 11. Their highestpowers are 2^2, 3^2, 5^1, 7^1 and 11^1.
Therefore, LCM = $2^2 \times 3^2 \times 5 \times 7 \times 11 = 13860$.

Example

Find the LCM of 56, 70 and 98.

$56 = 2^3 \times 7$

$70 = 2 \times 5 \times 7$

$98 = 2 \times 7^2$

The LCM = $2^3 \times 5 \times 7^2 = 1960$.

Example

Find the LCM and HCF of 630 and 150

Solution

$630 = 2 \times 3^2 \times 5 \times 7$

$150 = 2 \times 3 \times 5^2$.

The common prime factors with the least power are 2, 3 and 5.

Therefore, the HCF = $2 \times 3 \times 5 = 42$

To find the LCM;

$630 = 2 \times 3^2 \times 5 \times 7$

$150 = 2 \times 3 \times 5^2$

The prime factors available are 2, 3, 5 and 7. Their highest powers are 2^1, 3^2, 5^2 and 7^1.
Multiplying them gives the LCM.
So LCM is $2 \times 3^2 \times 5^2 \times 7 = 3150$.

Alternative method for finding the LCM:

Example

Find the LCM of 16, 12 and 24.

	16	12	24
2	8	6	12
2	4	3	6
2	2	3	3
2	1	3	3
3	1	1	1

Therefore, LCM of 16, 12 and 24 = $2^4 \times 3 = 48$

Alternative method for finding the HCF:

Example

Find the HCF of 12 and 15.

Solution

$F_{12} = \{1, 2, 3, 4, 6, 12\}$ and

$F_{15} = \{1, 3, 5, 15\}$

The common factors are {1, 3}. The highest of these is 3. Therefore, the HCF of 12 and 15 is 3.

Example

Find the H.C.F of 30 and 45.

Solution

$30 = 2 \times 3 \times 5$

$45 = 3 \times 3 \times 5$

Both 30 and 45 have 3×5 in common; thus the H.C.F of 30 and 45 is 15.

EXERCISE

State the LCM and HCF of

1. $2 \times 3,\ 2^2 \times 3 \times 5$
2. $3 \times 7,\ 2^2 \times 7$
3. $2 \times 11^2,\ 3 \times 11$
4. $2^2 \times 7,\ 2^3 \times 5^2 \times 7,\ 2^2 \times 7 \times 13$
5. $3^4,\ 32 \times 5^2$

Find the HCF and the LCM, leaving the LCM in prime factors.

6. 72, 162 7. 126, 198

8. 210, 336, 294. 9. 455, 286.

10. 616, 2156.

11. Five small containers of capacity 16, 72, 12, 24 and 56 litres are to be used to fill a bigger container. What is the capacity of the bigger container which can be filled by each of the above containers exactly without remainder when used separately?

12. Musa, John and David start at the same time, position and direction to run round a circular field. Musa takes 180 seconds, John takes480 seconds and David takes 720 seconds to complete one circuit. If they start running at 3.00 pm, at what time will they all be at the same position?

13. Find the value of n if $79_{13} = 144_n$.

14. Find the value of n if $124_n = 52_{ten}$

15. Express $\frac{108}{28}$ as a product of its prime factors.

16. A room measures 540 cm by 420 cm. Find the length of the largest square tiles that can be used to cover the floor without requiring any cutting.

17. Traffic lights at three different junctions show green light at intervals of 10 seconds, 12 seconds and 15 seconds. They allshow green at 1.00 p.m. At what time will they all again show green together?

18. In a large school, it is possible to divide the pupils into groups of equal numbers of 24, 30 or 32 and have no pupils left over.Find the least number of pupils in the school that makes this possible.

19. Find the shortest length that can be cut into exactly equallengths of 4 cm or 7 cm or 18 cm.

20. Find the LCM of the following groups of numbers. Leave the answers in power form.

(a)3, 9, 15 (b)4, 5, 6 (c)28, 36

(d)48, 90, 125 (e)200, 350

(f)18, 42, 84 (g)7, 75, 150

(h)30, 56, 72

Divisibility test

(i) A number is divisible by 2 if its last digit is even e.g. 136, 1760.

(ii) A number is divisible by 3 if the sum of its digits is divisible by3, e.g. 4713 since $4 + 7 + 1 + 3 = 15$ which is divisible by 3.

(iii) A number is divisible by 4 if the number formed by the last two digits is divisible by 4, e.g. 144; 128.

(iv) A number is divisible by 5 if its last digit is 5 or 0.

(v) A number is divisible by 6 if it is divisible by 2 and 3.

(vi) A number is divisible by 8 if the number formed by the last 3 digits is divisible by 8. e.g. 29848; 1048.

(vii) A number is divisible by 9 if the sum of its digits is divisible by 9 e.g. 447129 since 4+4+7+1+2+9 = 27 which is divisible by 9.

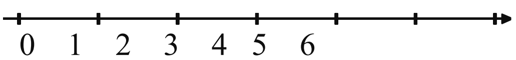
(viii)A number is divisible by 11 if the difference between the sum of the digits in even places and the sum of digits in odd places is 0 or divisible by 11. e.g., 733689
Digits in even places (3+6+9 = 18)
Digits in odd places (7+3+8 = 18).
The difference is 0. Hence 733689 is divisible by 11.
What about 80927?
Digits in even places are (0+2 = 2)
Digits in odd places are (8+9+7 = 24)
The difference is 22 which is divisible by 11. Hence 80927 isdivisible by 11.

Integers

This is a set of whole numbers together with negative wholenumbers. We can write
{*Integers*} = {...-4,-3,-2,-1,0,1,2,3,.....}
When numbers are used with symbols + or – in front of them, they are called **directed** numbers. E.g. +1, +2, +3, …-1, -2, -3, …

The number line

Natural numbers can be represented on a number line as shown inthe figure below;



On the number line, numbers to the left of 0 are less than 0 and are called **negative numbers**. Those to the right of 0 are greater than 0and are called **positive numbers**.

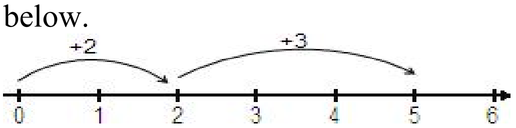
Addition of integers

Example

Use the number line to find the value of:
(a) $^{+}2 + ^{+}3$
(b) (-3)+ (-4)
(c) 7+(-5)

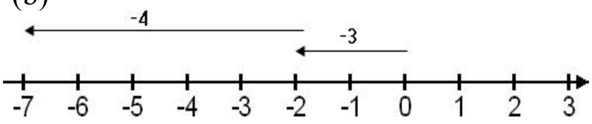
Solution:

(a) On the number line we move 2 steps from 0 to +2 and thenfrom this point we move a further 3 steps to the right to represent +3. We end up at +5. See figure below.

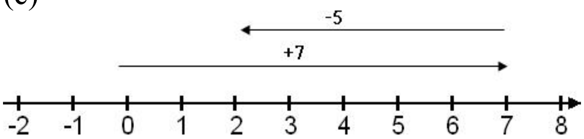


This shows that $^{+}2 + ^{+}3 = ^{+}5$.

(b)



(-3) + (-4) = -7 or -3 – 4 = -7
(c)



7+ (-5) = +2 or 7+ (-5) = 7 – 5 = 2
On the number line, move 7 steps to the right of 0 to represent +7 and then 5 steps to the left of +7 to represent -5. The end point is +2.

Subtracting integers

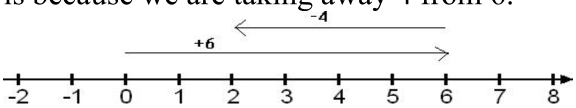
Example

Use a number line to find the value of:

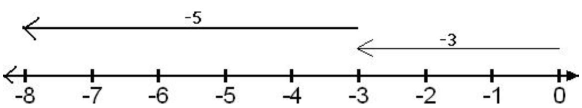
(a) +6 – (+4) (b) -3 – (+5) (c) 4 – (-3)

Solution

(a) In the figure below, we move 6 steps to the right of 0 to represent +6, and then from this point we move 4 steps to the left. This is because we are taking away 4 from 6.



We see that 6 – 4 = 2. Notice that (+6) – (+4) = (+6) + (-4) = 2. This means that subtracting a positive number is the same as adding anegative number.
(b) In the figure below we move 3 steps to the left of 0 to represent -3 and then from this point we move another 5 steps to the left.
Thus, -3 – (+5) = -8. Again, note that, -3 – (+5) = (-3) + (-5) = -(3 + 5) = -8.



(c) In the figure below, to subtract -3 from 4 we look for a number which when added to -3 gives 4. Therefore, we look for the number of steps between 4 and -3. Since (+4) = (+7) + (-3), it follows that (+4) – (-3) = (+7). Thus, 4 – (-3) = 4 + 3 = 7. This means that subtracting a negative number is the same as adding the positive number.

Therefore, in general, if a and b are numbers, then,
(i) a – (+b) = a – b
(ii) a – (-b) = a + b

EXERCISE
Give the value of:
1. (+6) – (+2) **2**. (-2) – (+5) **3**. +3 + (-6)
4. (-2) – (-7) **5**. 6 + (-2) **6**. (-9) + (-1)
7. (-6) + 6 **8**. 0 – 4 **9**. 300 – 500
10. 0 – (-6) **11**. 18 – (-20) **12**. -43 – (-43)
13. (+2) – (-3) **14**. 0 - 58
Simplify:
15. (+5a) – (+a)
16. (-2b) – (-3b)

Multiplying integers

When the same number is added to itself a number of times, the result obtained is the same as multiplying the number by the number of times it is added. For example:
(i) 3 +3 + 3 +3 +3 = 15
$3 \times 5 = 15$

(ii) (-4) + (-4) + (-4) = -12
$-4 \times 3 = -12$

However, this relationship between addition and multiplication doesnot apply in cases where two negative numbers are multiplied.

Consider the following multiplication table.

$4 \times -2 = -8$
$3 \times -2 = -6$
$2 \times -2 = -4$
$1 \times -2 = -2$
$0 \times -2 = 0$
$-1 \times -2 = +2$
$-2 \times -2 = +4$
$-3 \times -2 = +6$
$-6 \times -2 = +8$

You will notice that the numbers in the first column decrease by 1 as you go down the table whist the corresponding products increase by 2.
The numbers in the first column are all multipliedby the same number (-2).

Whenever two negative numbers are multiplied,the product is a positive number.

In general the **rules of multiplication** of are:

- Two numbers with like signs have a positive product.
- Two numbers with unlike signs have a negative product.
- Multiplication of any number by zero always gives zero.

Dividing integers

When a number is divided by another number, the result obtained iscalled a **quotient**. For example, the quotient of $12 \div 6$ is 2. The rulesof dividing integers are the same as those of multiplying except that we do not divide numbers by zero. It is meaningless to write $5 \div 0$ because the quotient does not exist. However, zero divided by any number is always zero, for example,
$0 \div (-42) = 0 \div 1000 = 0$
Summary for the rules of multiplication and division of integers

RULE	RESULT	EXAMPLE
A positive × A positive	A positive	$^{+}2 \times ^{+}4 = {}^{+}8$
A positive × A negative	A negative	$^{+}2 \times ^{-}3 = {}^{-}6$
A negative × A negative	A positive	$^{-}2 \times ^{-}2 = {}^{+}4$
A positive÷A positive	A positive	$^{+}4 \div ^{+}2 = {}^{+}2$
A positive ÷ A negative	A negative	$^{+}4 \div ^{-}2 = {}^{-}2$
A negative ÷ A positive	A negative	$^{-}4 \div ^{+}2 = {}^{-}2$
A negative ÷A negative	A positive	$^{-}4 \div ^{-}2 = {}^{+}2$

Exercise

Find the values of:
1. $(-2) \times (+3)$ **2.** $(-10) \div (-2)$

3. (-5) × (-4) **4.** (+6) × (-5)
5. (-4) × (-2) **6.** (-6) ÷ (+3)
7. (+24) ÷ (+6) **8.** 0 × (-3)
9. (+3) × (+4) **10.**(-18) ÷ (-6)
11.(-7) ÷ (+7) **12.**(-3) × (-3)

Simplify:

13.(-4x) ÷ (+x) **14.**(-3x) × (-2x)
15.(-6y) ÷ (-2) **16.**(+ab) ÷ (-b) **17.**(-xy) ÷ (-x) **18.**(+18x) ÷ (-3x)
19.(-2) × (+3y) **20.**(-5) × (-2d)

Combined operations

When a question involves more than one operation, then there is a sequence of operations which has to be followed. The sequence is: Brackets, Of, Division, Multiplication, Addition, and Subtraction. Whenever there are brackets, the operations in the brackets must beperformed first followed by the others according to the above sequence. Division and multiplication have no priority over each other so they can be performed in the order they appear in the expression.

When addition and subtraction are the only operations in an expression, they can be carried out in the order in which they appear in the expression. For example:

10 + 5 – 3 = (10 + 5) – 3 = 15 – 3 = 12.
Or 10 + 5 – 3 = 10 + (5 – 3)
= 10 + 2 = 12.

6 – 4 + 1 = (6 + 1) – 4 = 7 – 4 = 3
Or 6 – 4 + 1 = (6 – 4) + 1
= 2 + 1 = 3

Example

Find the values of:

(a) 4 + 12 ÷ 6
(b) 3 + 20 × 2 - 12
(c) 39 ÷ (16 – 3) + 4 × 5 – 8
(d) 6 × 5 – 33 ÷ 3 + 40

Solutions

(a) 4 + 12 ÷ 6 = 4 + (12 ÷ 6)
= 4 + 2
= 6

(b) 3 + 20 × 2 – 12 = 3 + (20 × 2) - 12
= 3 + 40 – 12
= 43 – 12 = 31

(c) 39 ÷ (16 – 3) + 4 ×5 – 8
= 39 ÷ 13+ 4×5 – 8
= (39 ÷ 13) + (4 ×5) – 8
= 3 + 20 – 8 = 23 – 8 = 15

(d) 6 ×5 – 33 ÷ 3 + 40 = 30 – 11 + 40
= (30 + 40) – 1
= 70 – 11 = 59

Remember that we use brackets when carrying out any operation involving negative numbers.

For example, we write 8 × (-4) and not 8 ×- 4 or 5 + (-2) and not 5 + -2.
However, note that it is appropriate to leave out brackets when you use the raised negative sign. Thus we can write the above expressions as, 8 × $^-$4 or 5 + $^-$2.

Exercise

Work out:

1. 16 + 2 - 5
2. 7 + 1 - 3 + 2
3. 13 - 4 + 8
4. 11 + 3 × 3
5. 5 × 3 - 5
6. 4 × 8 - 8 × 3
7. 18 ÷ 6 - 3 × 4 - 2
8. 18 ÷ (6 - 3) × 4 - 2
9. 30 ÷ (-6) - 4
10. 5 - (-12) × 3 - 24 ÷ 6
11. 63 ÷ 3 - 42 ÷ (-7)
12. -3 × 23 + (-5) × (-1) - 8 × (-4)
13. 6 + 36 ÷ 9 + 14 ÷ 2 × 5
14. {9 + (-2) × (-15)} × (-2 + 7) ÷ 3

Place value

A digit have a different value in a number because of its position in a number. The position of a digit in a number is called its place value.

Total value

This is the product of the digit and its place value.

Example

✓ *Three hundred and forty five million, six hundred and seventy eight thousand, nine hundred and one.*

✓ *Seven hundred and sixty nine million, Three hundred and one thousand, eight hundred and fifty four.*

Billions

A billion is one thousands million, written as 1, 000, 000,000.There are ten places in a billion.

Example

What is the place value and total value of the digits below?

a.) 47,397,263,402 (place value 7 and 8).
b.) 389,410 ,000,245 (place 3 and 9)

Solution

a.) The place value for 6 is ten thousands. Its total value is 60,000.
b.) The place value of 3 is hundred billions. Its total value is 300,000,000,000.

Rounding off

When rounding off to the nearest ten, the ones digit determines the ten i.e. if the ones digit is 1, 2, 3, or 4 the nearest ten is the ten number being considered. If the ones digit is 5 or more the nearest ten is the next ten or rounded up.

Thus 641 to the nearest ten is 640, 3189 to the nearest is 3190.

When rounding off to the nearest 100, then the last two digits or numbers end with 1 to 49 round off downwards. Number ending with 50 to 99 are rounded up.

Thus 641 to the nearest hundred is 600, 3189 is 3200.

Example

Rounding off each of the following numbers to the nearest number indicated in the bracket:

a.) 473,678 (100) b.) 524,239 (1000)
c.) 2,499 (10)

Solution

a.) 473,678 is 473,700 to the nearest 100.

b.) 524,239 is 524,000 to the nearest 1000

c.) 2,499 is 2500 to the nearest 10.

Operations on whole Numbers

Addition

Example

Find out:

a.) 98 + 6734 + 348
b.) 6349 + 259 +7954

Solution

Arrange the numbers in vertical forms

a.)

```
   98
 6734
+ 348
 7180
```

b.)

```
  6349
   259
+79542
 86150
```

Subtracting

Example

Find: 73469 – 8971

Solution

```
  73469
-  8971
  64498
```

Multiplication

The product is the result of two or more numbers.

Example

Work out: 469 x 63

```
   469
  X 63
  1407 →469 x 3 =1407
+28140→469 x 60 =28140
 29547
```

Division

When a number is divided by the result is called the quotient. The number being divided is the divided and thenumber dividing is the divisor.

Example

Find: 6493 ÷ 14

Solution

We get 463 and 11 is the remainder

Note:

6493 = (463 x 14) + 11

In general,

dividend = quotient x division + remainder

Operation	*Words*
Addition	*sum* *plus* *added* *more than* *increased by*
Subtraction	*difference* *minus* *subtracted* *fromless than* *decreased by* *reduced by* *deducted from*
Multiplicatio	*product of* *multiply* *times twice* *thrice*
Division	*Quotient* *of* *Divided* *by*
Equal	*Equal* *to* *Result* *is*

Word problem

In working the word problems, the information given must be read and understood well before attempting the question.

The problem should be broken down into steps and identify each other operations required.

Example

Otego had 3469 bags of maize, each weighing 90 kg. He sold 2654 of them.

a.) How many kilogram of maize was he left with?

b.) If he added 468 more bags of maize, how many bags did he end up with?

Solution

a.) One bag weighs 90 kg.

3469 bags weigh 3469 x 90 = 312,210 kg

2654 bags weigh 2654 x 90 = 238,860 kg

Amount of maize left= 312,210 – 238,860

= 73,350 kg.

b.) Number of bags = 815 + 468

=1283

Even Number

A number which can be divided by 2 without a remainder E.g. 0,2,4,6 0 or 83600, 7800, 806, 78346

Odd Number

Any number that when divided by 2 gives a remainder. E.g. 471,123, 1197,7129.The numbers ends with thefollowing digits 1, 3, 5,7 or 9.

Prime Number

A prime number is a number that has only two factors one and the number itself.For example, 2, 3, 5, 7, 11, 13, 17 and 19.

Note:

i.) 1 is not a prime number.

ii.) 2 is the only even number which is a prime number.

Questions

1.) Write 27707807 in words

2.) All prime numbers less than ten are arranged in descending order to form a number

a.) Write down the number formed

b.) What is the total value of the second digit?

c.) Write the number formed in words.

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