P425/1
PURE MATHEMATICS
Paper 1
Nov. / Dec. 2020
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. Solve the equation: $\sin x + \sin 2x + \sin 3x = 0$ for $0^\circ \leq x \leq 180^\circ$. *(05 marks)*

2. (a) Express $Z = \frac{3+i}{1-i}$ in the form $a + bi$, where a and b are integers. *(03 marks)*

   (b) Find the argument of Z. *(02 marks)*

3. Given that $y = ln\left\{ x\sqrt{(x+1)^3} \right\}$, find $\frac{dy}{dx}$. *(05 marks)*

4. A plane is perpendicular to the vector $\boldsymbol{r} = (\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k})$ and contains the point $P(-2, 0, 4)$. Determine the equation of the plane. *(05 marks)*

   r- OA + MAB + λAC. 2x+ 3y+z = 6.

5. Evaluate $\int_0^{\pi/3} tan^2 \, {}^1\!/_2 x \, dx$. *(05 marks)*

6. In how many ways can the letters of the word BUNDESLIGA be arranged if;

   (a) there is no restriction? *(02 marks)*

   (b) the vowels must be together? *(03 marks)*

7. (a) Show that the curve whose parametric equations are $x = 9\cos\theta$ and $y = 12\sin\theta$ represents an ellipse. *(03 marks)*

   (b) Determine the eccentricity of the ellipse. *(02 marks)*

8. Find the gradient of the curve $x^2 \tan x - xy - 2y^2 = -2$ at the point $(0,1)$. *(05 marks)*

SECTION B: (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

(a) A polynomial $P(x)$ is given by $P(x) = (x+2)(x-1)Q(x) + (ax+b)$ where $Q(x)$ is the quotient and $ax + b$ is the remainder. When $P(x)$ is divided by $x - 1$, the remainder is 4 and when it is divided by $x + 2$, the remainder is 1. Find the values of a and b. *(05 marks)*

(b) (i) Expand $(1+x^4)^{-1/2}$ up to the fourth term.

(ii) Use the first two terms of the expansion to find the value of

$$\frac{1}{\sqrt{144.0144}}$$

correct to **two** significant figures. (*07 marks*)

10. A circle passes through the points (1, 3), (2, 2) and (5, 7).

Find the equation of the;

(a) circle. (*07 marks*)

(b) tangent to the circle at the point (1, 3). (*05 marks*)

11. Express $\dfrac{2-x+x^2}{(1+x)(1-x)^2}$ in partial fractions.

Hence evaluate $\displaystyle\int_0^{1/2} \frac{(2-x+x^2)}{(1+x)(1-x)^2}\,dx$ correct to **three** decimal places.

(*12 marks*)

12. (a) Determine the angle between the vectors

$\boldsymbol{p} = \boldsymbol{i} + 9\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{q} = \boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$. (*05 marks*)

(b) The vector equations of two lines are $\boldsymbol{r}_1 = \boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k} + \lambda(-\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k})$ and $\boldsymbol{r}_2 = -2\boldsymbol{j} + 5\boldsymbol{k} + \mu(\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k})$.

Find the coordinates of the point of intersection of the two lines.

(*07 marks*)

13. Express $8\sin\theta - 15\cos\theta$ in the form $R\sin(\theta - \alpha)$ where R and α are constants.

Hence find the maximum value of $8\sin\theta - 15\cos\theta$ and the smallest positive value of θ at which the maximum occurs. (*12 marks*)

14. (a) Solve the simultaneous equations:

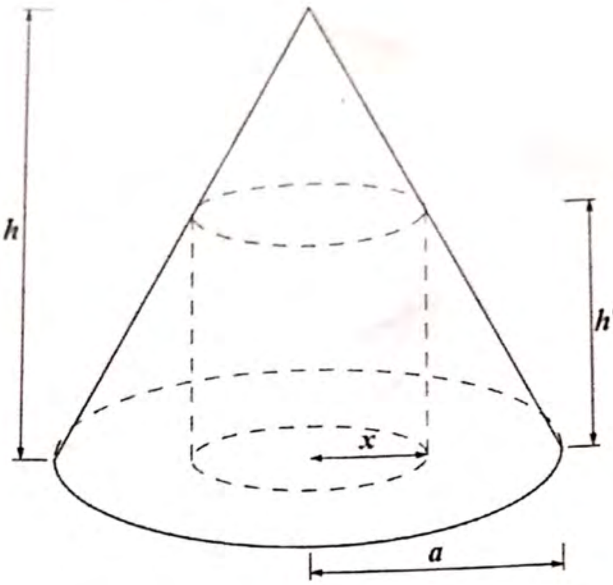
$$2x^2 - 5xy + 2y^2 = 0$$

$$x + y = 6$$

(*05 marks*)

(b) If α and β are the roots of the equation $x^2 - px + q = 0$, find the equation whose roots are $\dfrac{\alpha^2-1}{\alpha}$ and $\dfrac{\beta^2-1}{\beta}$.

15. The diagram below shows a cone of radius a and height h, in which a cylinder of radius x is inscribed.



(a) Express the height h' of the cylinder in terms of x, a and h. *(04 marks)*

(b) Show that $V' = \frac{4}{9} V$ where V' is the greatest volume of the cylinder that can be inscribed in the given cone of volume V. *(08 marks)*

16. (a) Solve the differential equation

$$\frac{ds}{dt} = \frac{2e^{2t}}{\sqrt{S}} \quad \text{given that } S = 9 \text{ when } t = 0.$$

(b) Determine the value of;

(i) t when $S = 16$

(ii) S when $t = 2.4$

(Give your answer to **2** significant figures) *(12 marks)*

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Squared paper is provided.

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Turn Over

SECTION A: (40 MARKS)

Answer **all** *questions in this section.*

1. Show that the modulus of $\frac{(1-i)^6}{1+i} = 4\sqrt{2}$. *(05 marks)*

2. Solve $2\cos 2\theta - 5\cos\theta = 4$ for $0^\circ \le \theta \le 360^\circ$. *(05 marks)*

3. Using the substitution $u = \tan^{-1} x$, show that $\int_0^1 \frac{\tan^{-1} x}{1+x^2}\, dx = \frac{\pi^2}{32}$. *(05 marks)*

4. Given the plane $4x + 3y - 3z - 4 = 0$;

(a) show that the point $A\ (1,1,1)$ lies on the plane. *(02 marks)*

(b) find the perpendicular distance from the plane to the point $B\ (1,5,1)$. *(03 marks)*

5. Find the equation of the tangent to the curve $y = \frac{a^3}{x^2}$ at the point $P\left(\frac{a}{t}, at^2\right)$. *(05 marks)*

6. Given that $\alpha + \beta = \frac{-1}{3}$ and $\alpha\beta = \frac{2}{3}$, form a quadratic equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. *(05 marks)*

7. Find the area enclosed between the curve $y = 2x^2 - 4x$ and the x– axis. *(05 marks)*

8. Given that $Q = \sqrt{80 - 0.1P}$ and $E = \frac{-dQ}{dP} \cdot \frac{P}{Q}$, find E when $P = 600$. *(05 marks)*

SECTION B: (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. (a) Determine the perpendicular distance of the point (4, 6) from the line $2x + 4y - 3 = 0$. *(03 marks)*

(b) Show that the angle θ, between two lines with gradients λ_1 and λ_2 is given by $\theta = \tan^{-1}\left(\frac{\lambda_1 - \lambda_2}{1 + \lambda_1\lambda_2}\right)$.

Hence find the acute angle between the lines $x + y + 7 = 0$ and $\sqrt{3}x - y + 5 = 0$. *(09 marks)*

10. (a) Given that $26\left(1 - \frac{1}{26^2}\right)^{\frac{1}{2}} = a\sqrt{3}$, find the values of a.

(05 marks)

(b) Solve the simultaneous equations:

$$2x = 3y = 4z,$$
$$x^2 - 9y^2 - 4z + 8 = 0.$$

(07 marks)

11. Express $7\cos 2\theta + 6\sin 2\theta$ in the form $R\cos(2\theta - \alpha)$, where R is a constant and α is an acute angle.

Hence solve $7\cos 2\theta + 6\sin 2\theta = 5$ for $0° \le \theta \le 180°$. *(12 marks)*

12. (a) Given that $y = ln\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$, show that $\frac{dy}{dx} = \frac{x^2 - 1}{x^2 - 4}$.

(05 marks)

(b) Evaluate $\int_0^4 \frac{dx}{x^2\sqrt{(25 - x^2)}}$. *(07 marks)*

13. Four points have coordinates A (3,4,7), B (13,9,2), C (1,2,3) and D (10,k,6). The lines AB and CD intersect at P. Determine the;

(a) vector equations of lines AB and CD. *(06 marks)*

(b) value of k. *(04 marks)*

(c) coordinates of P. *(02 marks)*

Turn Over

14. Expand $\sqrt{\left(\frac{1+2x}{1-x}\right)}$ upto the term in x^2.

Hence find the value of $\sqrt{\left(\frac{1.04}{0.98}\right)}$ to **four** significant figures. (*12 marks*)

15. (a) Differentiate $y = 2x^2 + 3$ from first principles. (*04 marks*)

(b) A rectangular sheet is 50 cm long and 40 cm wide. A square of x cm by x cm is cut off from each corner. The remaining sheet is folded to form an open box. Find the maximum volume of the box. (*08 marks*)

16. (a) Find $\int \frac{ln x}{x^2} dx$. (*04 marks*)

(b) Solve the differential equation

$\frac{dy}{dx} + y \cot x = x$, given that $y = 1$ when $x = \frac{\pi}{2}$. (*08 marks*)

END

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- Clearly indicate the questions you have attempted in a grid on your answer scripts.

<table>
<tr><td colspan="2">Question</td><td>Mark</td></tr>
<tr><td colspan="2">Section A</td><td></td></tr>
<tr><td rowspan="5">Section B</td><td></td><td></td></tr>
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<tr><td colspan="2">Total</td><td></td></tr>
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SECTION A

1. If $x = \log_a bc,\ y = \log_b ac\ and\ z = \log_c ab.$ Prove that;
$x + y + z = xyz - 2.$

2. Given that $y = tan\ xy$, show that
$\frac{dy}{dx} = \frac{y}{cos^2xy - x}$.

3. Prove that the $\int_o^{In2} \frac{e^x}{1+e^{2x}}\, dx = \tan^{-1}\frac{1}{3}$

4. The distance of the centre of the circle of radius 5 from the line $3x = 4y\ is$ 3 units. Find the equation of the tangent to the circle which is parallel to the line $3x = 4y$.

5. Show that the line $\frac{x-2}{2} = \frac{2-y}{1} = \frac{z-3}{3}$ is parallel to the plane 4x – y – 3z = 4, and find the perpendicular distance of the line from the plane.

6. Find x if $tan^{-1}x + tan^{-1}\frac{1}{3} = \frac{\wedge}{4}$.

7. The expression $ax^4 + bx^3 - x^2 + 2x + 3$ has a remainder $3x + 5$ when it is divided by x^2-x-2, find values of a and b.

8. If $y = \sqrt{(5x^2 + 3)}$, show that $\frac{yd^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 5.$

SECTION B
Attempt any 5 questions ONLY

9. a) Solve the inequality;
$\frac{x}{x+1} \leq \frac{x+2}{x+4}$
b) Given that f(x) = $\frac{sin^{-1}x}{\sqrt{(1-x^2)}}$, show that $(1 - x^2)f''(x) - 3xf^1(x) = f(x)$. Hence find the first two non-vanishing terms of the maclaurin's expansion.

10. a) Find $b\int_{3tan^{-1}4}^{4tan^{-1}3} \frac{cos\frac{x}{2}}{4-5sin\frac{x}{2}}dx$, give your answer to 2 decimal places.

11. (i) Show that In2^r for r = 1,2,3,………. is an arithmetic progression.
(ii) Find the sum of the first 10 terms of the progression.
(iii) Determine the least value of m for which the first 2m terms exceeds 883.7.

12. a) A tangent from the point T(t^2,2t) touches the curve y^2 = 4x. Find

i) The equation of the tangent
ii) The equation of the L parallel to the Normal at (t^2, 2t) and passing through (1,0).
iii) The point of intersection of the line L and the tangent.
b) A point P(x, y) is equidistant from x and T. show that the locus of
t^2 -3t – 2$(x + y)$ = 0

13. a) Without using tables, evaluate,
$\sin\left[\cos^{-1}\frac{4}{5} + 2\tan^{-1}\frac{-1}{2}\right]$
b) $sin3x + \frac{1}{2} = 2cos^2x\ fro\ 0\ \leq x\ \leq 2\ \barwedge$

14. a) Find the Cartesian equation of the plane containing the points A(2,-1,1) B(1,-2,0) and C(-3,6,1). Find the angle between this plane and the line;

$$\frac{x}{4} = \frac{y-1}{1} = \frac{z+3}{5}$$

b) The position vector of points **A** and **B** are $3\underset{\smile}{\boldsymbol{i}} - \underset{\smile}{8}\mathbf{j} + \underset{\smile}{\boldsymbol{k}}$ and $4\underset{\smile}{\boldsymbol{j}} - 2\underset{\smile}{\boldsymbol{k}}$ respectively. Find the position vector of the foot of the perpendicular from the origin 0 to the line **AB**.

15. a) If (1 +3i)$\mathbf{Z}_1$ = 5(1+i). Show that the locus of $|\mathbf{z} - \mathbf{z}_1|$ is a circle. Find the coordinates of the centre and radius of the circle.
b) Given that x and y are real, find the values of x and y which satisfy the equation.
$\frac{2y+4i}{2x+y} - \frac{y}{x-i} = 0$

16. a) Solve the differential equation
$sinx\frac{dy}{dx} + 2ycosx = 1$

b) An electric Kettle Switches itself off when the temperature of water in it reaches 100ºC at 11:00am when Mr. Nsamba came back and found the temperature of water to be 45ºC. 20 minutes later he measured it again and found it to be 65ºC. According to the law of heating, the rate of heating of a body in air is proportional to the excess temperature over the surrounding at any time t. if the surrounding temperature was 25ºC, Mr Nsamba wants to know the time when the kettle switched off itself.

END

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POST MOCK SET 1 2020

PURE MATHEMATICS

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Section A		
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SECTION A (40 MARKS)
Attempt all questions from this section.

1. If $\frac{\mathbf{2+\sqrt{2}}}{\mathbf{2-\sqrt{2}}} + \frac{\mathbf{1-\sqrt{2}}}{\mathbf{1+\sqrt{2}}} = \boldsymbol{a} + \boldsymbol{b}\sqrt{\mathbf{2}}$ Find the values of a and b. (5 marks)

2. The ninth term of an arithmetic progression is twice the third term, and the fifteenth term is 27. Evaluate the sum of the first 25 terms of the series. (5 marks)

3. Differentiate $x^{\cos x}$ with respect to x. (5 marks)

4. Evaluate the definite integral $\int_0^1 x \tan^{-1} x \, dx$(5 marks)

5. Solve the equation $3\cos 2\theta - 7\cos\theta - 2 = 0$for $0° \leq \theta \leq 360°$. (5 marks)

6. Find the equation of the circle which touches the line $3x - 4y = 3$ at the point $(5, 3)$ and passes through the point $(-2,4)$. (5 marks)

7. The roots of the equation $x^2 + px + 7 = 0$ are $\propto$ and β. Given that $\propto^2 + \beta^2 = 22$,
   find the possible values of p. (5 marks)

8. Prove that $\log_a x = \frac{1}{\log_x a}$. Hence solve the equation $\log_{10} x + \log_x 100 = 3$
   (5 marks)

SECTION A (60 MARKS)

Answer any five questions from this section.

9. (a) If $z = x + iy$, determine the Cartesian equation of the locus given by

$$\left|\frac{(z-1)}{(z+1-i)}\right| = \frac{2}{5}$$ (6 marks)

(b) Sketch the loci defined by the equations:

(i) $\arg(z+2) = \frac{-2\pi}{3}$

(ii) $\arg\left(\frac{z-3}{z-1}\right) = \frac{\pi}{4}$

(6 marks)

10. (a) Prove that $\sin 4\theta = \frac{4\tan\theta(1-\tan^2\theta)}{(1+\tan^2\theta)}$ (6 marks)

(b) Solve the equation $\tan^{-1}(1+x) + \tan^{-1} 1 - x = \frac{\pi}{4}$ (6 marks)

11. Find the coordinates of any maxima, minima and points of inflexion of the function $y = \frac{3x-1}{(4x-1)(x+5)}$ that it may have. Hence sketch the curve $y = \frac{3x-1}{(4x-1)(x+5)}$

(12 marks)

12. (a) Find $\int x\sqrt{(1-x^2)}\, dx$

(b) Express $\int_0^1 \frac{x^2+x+1}{(x+1)(x^2+1)} dx = \frac{3}{4}\ln 2 + \frac{\pi}{8}$ (9 marks)

13. (a) Find the particular solution of the differential equation $xy\frac{dy}{dx} = x^2 + y^2$, Given that $y = 2$, when $x = 1$ (6 marks)

(b) A lump of radioactive substance is disintegrating. At time t days after it was first observed to have the mass of 10 grams and $\frac{dm}{dt} = -km$ where k is a constant.Find the time, in days for the substance to reduce to 1 gram in mass, given that its half –life is 10 days. (The half – life is the time in which half of any mass of the substance will decay.) (6 marks)

14. (a) Find the values of m for which the line $y = mx$ is a tangent to the circle $x^2 + y^2 + fy + c = 0$(3 marks)

(b) Find the points where the line $2y - x + 5 = 0$ meets the circle $x^2 + y^2 - 4x + 3y - 5 = 0$Obtain the equation of the tangents and normal to the circle at these points(6 marks)

15. (a) Show that the points A,B and C with position vectors $2\hat{\imath} + 3\hat{\jmath}, 4\hat{\imath} + 5\hat{\jmath}, 6\hat{\imath} + 9\hat{\jmath}$ respectively are the vertices of a triangle. Find the area of the triangle. (5 marks)

(b) Find a vector $r$perpendicular to the vectors $s = 5\hat{\imath} + 3\hat{\jmath} + k$ and $t = -\hat{\imath} + 3\hat{\jmath} + 2\mathrm{k}$.
Hence, find the equation of a plane passing through the point $A(5, -1, -2)$ and
parallel to s and t. Find the angle between the plane and the line $\frac{x-2}{1} = \frac{y-2}{2} = \frac{z-2}{3}$(7 marks)

16. (a) If $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ show that $\frac{dy}{dx} = \frac{1}{1+x^2}$ (6 marks)

(b) Use the Maclaurin's theorem to find the first four terms of the expansion of $e^x \sin x$. (6 marks)

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POST MOCK SET 3 2020

PURE MATHEMATICS

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SECTION A (40 MARKS)

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1. In a triangle ABC, $\overline{AB} = x - y$, $\overline{BC} = x + y$, and $\overline{CA} = x$. Show that $\cos A = \dfrac{x - 4y}{2(x - y)}$. (5 marks)

2. If the line $3x - 4y - 12 = 0$ is the tangent to the circle with centre at (1,1). Find equation of the circle. (5 marks)

3. Using the substitution $u = \sqrt{x}$, show that $\int_1^4 \dfrac{dx}{x + \sqrt{x}} = ln\left(\dfrac{9}{4}\right)$. (5 marks)

4. By using the Binomial theorem, expand $(25 + x)^{1/2}$ as far as the term in x^2. Hence evaluate $\sqrt{26}$ correct to 3 decimal places. (5 marks)

5. The points A, B, C and D have coordinates $(-7,9), (3,4), (1,2) and (-2,-9)$ respectively. Find the vector equation of the line PQ where P divides AB in the ratio 2:3 and Q divides CD in the ratio 1:-4. (5 marks)

6. A committee of four people is to be selected from a group of nine people. In how many ways can a committee be chosen if two particular people agree to serve only if both are selected? (5 marks)

7. If $y = ae^x + b\cos x$, show that;
$(1 + \tan x)\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + (1 - \tan x)y = 0$ (5 marks)

8. Find the volume generated if the area between the curve $y = 16 - x^2$, the y-axis and the line $y = 6x$ is rotated through four right angles about the x-axis. (5 marks)

SECTION B (60 MARKS)

Answer only five questions from this section.

9. (a) Given that $z_1 = 3 + 2i$ and $z_2 = 2 - i$. Find $z_1 + z_2$,graphically. (5 marks)

   (b) If $z = x + iy$ is a complex number, describe and illustrate on the Argand diagram the locus of $\left|\frac{z+2}{z}\right| = 3$ (7 marks)

10. (a) Solve for θ, in the equation $2\cos^2\left(\theta - \pi/2\right) - 3\cos\left(\theta - \pi/2\right) - 2 = 0$ where $0^0 \le \theta \le 360^0$ (5 marks)

    (b) If $\tan\alpha = p$, $\tan\beta = q$, $\tan\gamma = r$, prove that $\tan(\alpha + \beta + \gamma) = \frac{p + q + r - pqr}{1 - pr - rq - pq}$.

    hence, show that $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{\pi}{4}$.

    (7 marks)

11. Find the values of A, B, C and D such that $\frac{2x^3 - 1}{x^2(2x-1)} = A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{(2x-1)}$.

    Hence show that $\int_1^2 \frac{2x^3 - 1}{x^2(2x-1)}dx = \frac{3}{2} + In\left(\frac{4}{\sqrt{27}}\right)$. (12 marks)

12. (a) The points P and Q have position vectors $\underset{\sim}{\mathbf{p}}$ and $\underset{\sim}{\mathbf{q}}$ respectively with respect to an origin o. Point R lies on PQ and $\frac{PR}{PQ} = \lambda$, show that $\underset{\sim}{\mathbf{r}} = (1-\lambda)\underset{\sim}{\mathbf{p}} + \lambda\underset{\sim}{\mathbf{q}}$ (4 marks)

    (b) (i) Find the coordinate of the point P in which the plane $4x + 5y + 6z = 87$ intersects the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z+4}{2}$

    (ii) Determine the angle between the line and the plane in b (i) (8 marks)

13. (a) If the line $y = 2x + c$ is a tangent to the hyperbola $\frac{x^2}{6} - \frac{y^2}{4} = 1$ show that $c = \pm 2\sqrt{5}$ (5 marks)

(b) A point P on the curve is given parametrically by $x = 3 - \cos\theta$ and $y = 2 + \sec\theta$. Find the equation of the normal to the curve at the point where $\theta = \pi/3$ (7 marks)

14. The first, fourth and eighth terms of an arithmetic progression (A.P) form a geometric progression (G.P). If the first term is 9, find the
(a) common difference of the AP (4 marks)
(b) common ratio of the G.P (2 marks)
(c) difference in sums of the first 6 terms of the progressions. (6 marks)

15. (a) Differentiate the following with respect to x and simplify your results.
(i) $y = 3^{x}$
(ii) $y = \sin^4 x + \cos^4 x$ (6 marks)

(b) Find the coordinates of the turning point on the curve $y = \frac{\log_e x}{x}$ and identify the nature. (6 marks)

16. (a) Solve the differential equation
$x^2 \frac{dy}{dx} + xy = 2 + x^2$, given $y(1) = 2$ (5 marks)

(b) A body of unit mass falls under gravity in a medium in which the resistance R is proportional to the velocity, V of the body. If the body was initially at rest, show that the velocity after time t is given by, $V = gk^{-1}(1 - e^{-kt})$, where k is a constant and g is the acceleration due to gravity. (7 marks)

END

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Question		Mark
Section A		
Section B		
Total		

SECTION A (40 MARKS)

1. Solve the simultaneous equations;
$4^x = 6 \ and \ 2(2^x) = 3(2^y)$.

2. When a polynomial $P(x)$ is divided by $x^2 - 7x - 18$, the remainder is 2x + 5. Find the remainder when p(x) is divided by;
(i) $x - 9$ (ii) $x + 2$

3. Show that; $\cot^{-1}\left(\frac{5}{2}\right) + \cot\left(\frac{9}{5}\right) = \cot^{-1}\left(\frac{35}{43}\right)$

4. Find the equation of the Normal to the circle $x^2 + y^2 - 2x - 24 = 0$ at the point (0, 7)

5. Evaluate; $\int_0^{\pi/3} \frac{\sin x}{(1+cosx)^2}\, dx$

6. The fourth, seventh and sixteenth terms of an A.P are in a G.P. If the first six terms of the A.P have a sum of 12. Find the common difference of the A.P and the common ratio of the G.P.

7. Find the position vector of the point of intersection, P, of the line with equation;
$\boldsymbol{r} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and the line joining the points with position vectors $\begin{pmatrix} 2 \\ 4 \end{pmatrix} \ and \ \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

8. Find $\frac{dy}{dx}$ $in \ y = In\left\{e^x \left(\frac{x-2}{x+2}\right)\right\}^{\frac{3}{4}}$

SECTION B

9. a) Expand up to the 4th term of

(i) $(1 + x)^{-1}$

(ii) $(1-2x)^{\frac{1}{2}}$, and hence show that if x is small than

$(1+x)^{-1} - (1-2x)^{\frac{1}{2}} \approx \frac{3x^2}{2}$ (6marks)

b) Apply the maclaraurin's series to establish a series for; $In\,(1+x)$. hence, if $x = \frac{b}{a}$. Show that $\frac{b^2 - a^2}{2ab} = x - \frac{x^2}{2} + \frac{x^3}{2}$. (6marks)

10. a) Using the substitution $y = vx$, solve the differential equation.

$(2x - y)\,\frac{dy}{dx} = 2x + y$; given that $y = 3$ when $x = 2$. (5marks)

b) An electron survey revealed that during the parliamentary campaigns in a certain district, Mr. Katuntu was gaining support at a rate proportional to the product of the number of people already supporting him and those who were not yet supporting him. Po was the total population of the electorate and P was the number of people supporting him at a time t.

(i) Write a differential equation describing Mr. Katuntu's support.

(ii) If initially Mr. Katuntu had 100,000 supporters and the opinion polls revealed that he was gaining 5000 people per week, and the total electrorate being 2,000,000 people and that he is to win the election if he gets 51% of the vote, find how many weeks Mr. Katuntu needed to win the election. (7marks)

11. (a) Find the acute angle between the line $\frac{x-6}{5} = \frac{1-y}{1} = z + 1$ and the plane, 7x – y + 5z = -5, giving your answer to the nearest degree.

(b) Find the point where the line $x + 1 = \frac{y-2}{4} = z - 3$ *cuts the plane* $\underset{\smile}{r}\cdot\begin{pmatrix}2\\-1\\3\end{pmatrix} = 8$

(6marks)

12. A curve is given by $y = \frac{x-1}{(2x-1)\,(x+1)}$. Sketch the curve clearly starting the asymptotes and the restricted region for y. (12marks)

13. a) Show that 25x^2 + 9y^2 – 100x – 54y = 44. Represent an ellipse. State the co-ordinates of its;

(i) centre (ii) eccentricity (iii) Focii (7marks)

(b) A curve is given by the parametric equations X =4cos2t, y = 2sint. Show that they represent a parabola, state its vertex and sketch the curve. (5marks)

14. Resolve $\frac{x^3-3}{(x-2)\,(x^2+1)}$ into partial fractions. Hence evaluate; $\int_3^4 \frac{x^3-3}{(x-2)\,(x^2+1)}\,dx$ (12marks)

15. a) Differentiate with respect to x;

(i) $(cosx)^{sinx}$

(ii) $tan^{-1}\left[\frac{1-x}{1+x}\right]dx$

(12marks)

Simplest form. (8marks)

b) If $y = e^{4x}cos3x; shat\ that\ \frac{dy^2}{dx^2} - 8\frac{dy}{dx} + 25y = 0$ (4marks)

16. a) Prove that in any triangle ABC

$tanBcotC = \frac{a^2 + b^2 - c^2}{a^2 - b^2 + C}$ (6marks)

(b) If $Z = (1 + cos2\theta) + i(\sin 2\theta); where\ \frac{\bar{\wedge}}{2} \leq \theta \leq \frac{\bar{\wedge}}{2}$ prove that $|Z| = 2cos\theta.$

(6marks)

END

P425/1
PURE MATHEMATICS
PAPER 1
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

POST MOCK SET 6 2020

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

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<table>
<tr><td colspan="2">Question</td><td>Mark</td></tr>
<tr><td colspan="2">Section A</td><td></td></tr>
<tr><td rowspan="5">Section B</td><td></td><td></td></tr>
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<tr><td></td><td></td></tr>
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<tr><td></td><td></td></tr>
<tr><td colspan="2">Total</td><td></td></tr>
</table>

SECTION A (40 marks)

*(Answer **all** questions in this section)*

1. Solve for x, in the equation $9^{x-1} - 3^{x+2} + 162 = 0.$ ***(5 marks)***

2. The lines $4x - 3y = 5$ and $y = 3$ are tangents to two circles whose centres lie on the line $x = 7$. Find the distance between the centres of the circles. ***(5 marks)***

3. Solve $sec^2(2\theta) - 3tan2\theta + 1 = 0\,, \quad for \;\; 0^0 \leq \theta \leq 180^0$ ***(5 marks)***

4. The ages of a mother and her three children are in a geometrical progression, the sum of their ages is 195 years and the sum of the ages of the two young children is 60 years. Find the age of the mother. ***(5 marks)***

5. Evaluate $\int_3^5 \frac{2(x+1)}{2x^2-3x+1}\, dx.$ ***(5 marks)***

6. The equation of the normal to the curve $xy^2 + 3y^2 - x^3 + 5y - 2 = 0$ at the point $(a, -2)$ *is* $15x - 8y - 46 = 0$. Find the value of a. ***(5 marks)***

7. Find $\frac{dy}{dx}$ if $y = xsin^2x$. when $x = \frac{\pi}{4}$ ***(5 marks)***

8. Find the Cartesian equation of a plane containing point (1, 3, - 4) and the line $\frac{x-1}{2} = \frac{y+2}{3} = z.$ ***(5 marks)***

SECTION B (60 marks)

(Answer any **five** questions from this section. All questions carry equal marks)

9. (a.) Given that $2A + B = 135$ show that $tanB = \frac{tan^2A - 2tanA - 1}{1 - 2tanA - tan^2A}$. ***(4 marks)***

(b.) If α is an acute angle and $tan\alpha = \frac{4}{3}$, show that

$4\sin(\theta + \alpha) + 3\cos(\theta + \alpha) = 5cos\theta$. Hence solve for θ the equation $4\sin(\theta + \alpha) + 3\cos(\theta + \alpha) = \frac{\sqrt{300}}{4}$ for $-180^0 \leq \theta \leq 180^0$. ***(8 marks)***

10. (a.) Show that $y = x - 3$ is a tangent to the curve $y = x^2 - 5x + 6$. ***(3 marks)***

(b.) A chord to the parabola $4x - 3y^2 = 0$ is parallel to the line $2x - y = 4$ and passes through point (1, 1). Find;

(i.) the equation of the chord.

(ii.) the coordinates of the points of intersection of the chord with the parabola.

(iii.) the acute angle between the chord and the directrix of the parabola.

(9 marks)

11. (a.) Expand $(4 - 3x)^{\frac{1}{2}}$ in ascending powers of x up to the term in x^3. Taking $x = \frac{1}{25}$ find $\sqrt{97}$. ***(8 marks)***

(b.) Find the term independent of x in the binomial expansion of $\left(2x - \frac{1}{x^2}\right)^9$.

(4 marks)

12. (a.) Solve for x and y values in the equation; $\frac{x}{2+3i} + \frac{y}{3-i} = \frac{6-13i}{9+7i}$. ***(6 marks)***

(b.) Given that $-4 + i$ is a root of the equation $z^4 + 6z^3 + 6z^2 + 6z + 65 = 0$, find the other roots of the equation and represent the roots in polar form. ***(6 marks)***

13. (a.) Find the volume of a solid generated by rotating about the y-axis, the area enclosed by the curve $y^2 + 4x = 9$, the y-axis and $y = -2$. ***(5 marks)***

(b.) Find $\int xln(2x)dx$. ***(3 marks)***

(c.) Evaluate $\int_0^1 \frac{2x-1}{(x-3)^2}\, dx$. ***(4 marks)***

14. The points A, B, C and D are given by the coordinates (5, 2, -3), (-1, 0, -1), (9, 5, -8) and (5, 7, -14) respectively. If lines AB and CD intersect at point E. Find;

(i.) Equations of lines AB and CD.

(ii.) Coordinates of point E

(iii.) The acute angle between lines AB and CD. ***(12 marks)***

15. A curve is given by the parametric equations; $x = 3t$ and $= \frac{2t^2}{1-t}$.

(a.) Find the Cartesian equation of the curve.

(b.) Sketch the curve, showing clearly the asymptotes and turning points.

(12 marks)

16. (a.) Solve the differential equation $\frac{dy}{dx} = 4x - 7$, given that $y(2) = 3$. ***(3 marks)***

(b.) The rate at which a candidate was losing support during an election campaign was directly proportional to the number of supporters he had at that time. Initially he had V_0 supporters and t weeks later, he had V supporters.

(i.) Form a differential equation connecting V and t.

(ii.) Given that the supporters reduced to two thirds of the initial number in 6 weeks, solve the equation in (i.) above.

(iii.) Find how long it will take for the candidate to remain with 20% of the initial supporters. ***(9 marks)***

P425/1
PURE MATHEMATICS
PAPER 1
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

POST MOCK SET 7 2020

PURE MATHEMATICS

Paper 1

3 hours

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Section A		
Section B		
Total		

SECTION A (40 MARKS)

1. If $y=e^{x}\sin x$, show that $\frac{d^2y}{dx^2}=2\left(\frac{dy}{dx}-y\right)$.

(5marks)

2. Given that $\log_{10}2=a$, prove that $\log_8 5=\frac{1-a}{3a}$. *(5marks)*

3. Solve by echelon method the following set of simultaneous equation:

$$x+3y+z=6$$
$$2x+y-4z=7$$
$$5x-6y+z=9$$

(5marks)

4. Partialise $f(x)=\frac{1}{(x+1)(x-3)}$ hence evaluate $\int f(x)\ dx$. *(5marks)*

5. Form the equation of the circle which passes through A(1, 3)B(3, 5) and C(5, 3) in the form $x^2 + y^2 + 2fx + 2gy + c = 0$ hence state its centre and radius. *(5marks)*

6. Solve the equation $4\cos x - 2\cos 2x = 3$, for $0^0 \leq x \leq 2\pi$ *(5marks)*

7(i) In how many ways can the letters of the word GEOMETRY be arranged in a row?

(ii) In how many of these arrangements are the two E's together? *(5marks)*

8. The second, fourth and eighth terms of an AP are in a GP. The sum of the third and fifth terms is 20. Determine the first three terms of the AP stated above.

(5marks)

SECTION B (60 MARKS)

9a) If x is small enough so that terms in x^3 and higher powers may be ignored, use binomial expansion to show that $\sqrt{\left(\frac{1-x}{1+2x}\right)}=1-\frac{3x}{3}+\frac{15x^2}{8}$. ***(8marks)***

b) Expand by use of Maclaurin series up to the term in x^2 the function $f(x)=\sin x$. Hence evaluate $\sin 30^0$ to 4 decimal places. ***(4marks)***

10a) Initially, the number of bacteria present in a culture solution is N_o.
At time t = 1hour, the number of bacteria is measured to be $\frac{3N_0}{2}$. The rate of growth is assumed to be proportional to the number of bacteria present at any time t. Show that the time necessary for the bacteria to grow to $3N_o$ (triple the original) is approximately 2.7hrs. *(6marks)*

b) A small metal piece initially at 20^0C is dropped into a large container of water kept at 100^0C. It was observed that the temperature of the metal increased by 2^0C in one minute.

(i) How long will it take for the temperature of the metal to increase to 90^0C?

(ii) Find the temperature of metal after 20minutes. *(6marks)*

11. Evaluate the following:

(i) $\int_0^{\pi/4} x^2 \sin 3x \quad dx$ *(4marks)*

(ii) $\int \frac{x^3}{16+x^8} dx$ *(4marks)*

(iii) $\int \cos^5 x \quad dx$ *(4marks)*

12a)(i) Form the equation of the plane perpendicular to line $\frac{x-3}{2} = \frac{y+1}{-5} = \frac{z-4}{2}$ passing through a point **A**(5, -6, 6). *(4marks)*

(ii) Determine the point **B** where the formed plane meets the line in (i) above. *(4marks)*

b) Determine the shortest distance from the point P(2, -5, 3) to the line $\frac{x-1}{4} = \frac{y+3}{1} = \frac{z-2}{-2}$. *(4marks)*

13a) Prove that $\frac{\sin 3A \sin 6A + \sin A \sin 2A}{\sin 3A \cos 6A + \sin A \cos 2A} = \tan 5A$. *(4marks)*

b) Show that $\tan^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \frac{\pi}{4}$. *(4marks)*

c) Solve for θ in the range $0 \le \theta \le 2\pi$ if $4\cos\theta + 3\sin\theta = 5$. *(4marks)*

14. Sketch the following curve systematically $y = \frac{3(x-3)}{(x+1)(x-2)}$. *(12marks)*

15a) Determine the square root of the complex number 15 + 8i. *(4marks)*

b) Solve the equation $z^4 + 6z^2 + 25 = 0$ *(4marks)*

c) Evaluate $\frac{\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^4}{\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)3}$ and give the solution in modulus-argument form.

(4marks)

END

P425/1
PURE MATHEMATICS
PAPER 1
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

POST MOCK SET 8 2020

PURE MATHEMATICS

Paper 1

3 hours

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Question		Mark
Section A		
Section B		
Total		

SECTION A: (40MARKS)

*Answer **all** the questions in this Section.*

1. Find the sum of the numbers between 5 and 250 which are exactly divisible by 4. *(5marks)*

2. Given that the line; $\frac{x-3}{4}=\frac{y-4}{-3}=\frac{z+3}{4}$ meets the plane $4x-3y-4z=3$ at M. Find the coordinates of M. *(5marks)*

3. Use the substitution $x = sin\theta$ to find the integral; $\int \frac{2x^3}{\sqrt{1-x^2}}\,dx$. *(5marks)*

4. Express $tan(45^0 + x)$ in terms of $tanx$. Hence prove that; $tan75^0 = 2 + \sqrt{3}$.

   (5marks)

5. Given $A(3,4)$ and $B(-2,3)$, find the equation of the locus of points $P(x,y)$ which divide AB in the ratio $2:1$. *(5marks)*

6. A women football team manager intends to take 18 players for a tournament. The manager has 2 goal keepers, 8 defenders, 4 mid fielders and 8 strikers. In how many ways can the team be chosen if it must contain both goal keepers, at least 3 midfielders and 7 strikers. *(5marks)*

7. Solve the differential equation; $Cosecx\frac{dy}{dx} = e^x cosecx + 3x$. *(5marks)*

8. Solve for x in the equation; $log_{(x+3)}(2x+3) + log_{(x+3)}(x+5) = 2$. *(5marks)*

SECTION B (60MARKS)

*Attempt any **five** questions from this Section.*

9. Given that $f(x) = \frac{x^3+2x^2+61}{(x+3)^2(x^2+4)}$, express $f(x)$ in partial fraction. Hence evaluate; $\int_0^1 f(x)dx$. *(12marks)*

10. $P(ap^2,\ 2ap)$ and $Q(aq^2,\ 2aq)$ are two variable points on the parabola $y^2 = 4ax$. If PQ subtends a right angle at the origin, prove that $pq = -4$.

a) Prove that PQ passes through a fixed point on the axis of the parabola.

b) The tangents at P and Q meet at R, find the equation of the locus of R. *(6marks)*

11. a) Differentiate $tan^{-1}\left(\frac{\sqrt{InX}}{e^{2x}}\right)$. *(6marks)*

b) Evaluate the integral; $\int_0^{\frac{\pi}{6}} \frac{2cos\theta+sin\theta}{cos\theta-sin\theta}\ d\theta$. *(6marks)*

12. a) P is the foot of the perpendicular from the point $A(1,1,1)$ to the line $\frac{X-1}{2} = \frac{y-1}{1} = \frac{Z-2}{1}$. Determine the perpendicular distance of A from the line to 4 $dp's$. *(5marks)*

b) Given the points $A(-1,2,3)$ and $P(2,3,4)$. If the point $B(a,2a,3)$ lies on the plane $2x - 3y + 4z + 8 = 0$. Find the value of a and the angle between AP and AB. *(7marks)*

13. a) Solve the equation $tan\theta - cot\theta = -1$ for $0^0 \leq \theta \leq 360^0$. *(5marks)*

b) Prove that $\frac{Sin3\theta}{1+2cos2\theta} = Sin\theta$. Hence show that $Sin15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$. (7marks)

14. a) Prove that $log_a^b = \frac{1}{log_b a}$. hence solve the equation $log_2 x + \ log_x 2 =$ 2.5. *(5marks)*

b) A polynomial is given by $P(x) = x^3 + Ax^2 + x - 6$. The ratio of the remainder when $P(x)$ is divided by $(X + 1)$ to the remainder when divided by $(x - 2)$ is $-1{:}\,5$. find the value of A. *(7marks)*

15. a) If $Z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$, express Z in modulus argument form. *(5marks)*

b) Use demoiver's theorem to prove that $2cos\theta = Z + \frac{1}{Z}$ then $2cosn\theta = Z^n + \frac{1}{Z^n}$. Hence solve the equation $5Z^4 - 11Z^3 + 6Z^2 - 11Z + 5 = 0$. *(7marks)*

16. a) Determine the nature of the turning points of the curve $y = x(1 - x)^2$.

(5marks)

b) The acceleration of a particle is proportional to 2t-3. If the velocity increases from 4ms^{-1} to 8ms^{-1} in the first 2 seconds of motion, find;

i) its initial acceleration *(5marks)*

ii) the velocity after 5 seconds. *(2marks)*

****** *END* ******

P425/1
PURE MATHEMATICS
PAPER 1
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

POST MOCK SET 9 2020

PURE MATHEMATICS

Paper 1

3 hours

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Total		

SECTION A (40 Marks)
Attempt ALL questions in this section.

1. Solve the inequality $\frac{1+x}{4+x} \geq \frac{5-2x}{x}$ (5marks)

2. Evaluate $\int_3^4 \frac{1}{x^2-3x+2}dx$ (5marks)

3. Solve the equation $2tan\theta + sin2\theta sec\theta = 1 + sec\theta$ for $0 \leq \theta \leq 2\pi$. (5marks)

4. The line $5x-2y+8=0$ is a tangent to the circle with centre at $(-2,3)$. Find the equation of the circle. (5marks)

5. Expand $(25-2x)^{\frac{1}{2}}$ in ascending powers of x up to the term in x^3. Hence by taking x=1, obtain the value of $\sqrt{23}$ correct to four significant figures. (5marks)

6. If $y = e^{2x}sin2x$, show that $\frac{d^2y}{dx^2} = 8(2e^{2x}cos^2x - 1)$. (5marks)

7. The position vectors of the points P and Q are $3\underline{i} - \underline{j} + 2\underline{k}$ *and* $2\underline{i} + 2\underline{j} + 3\underline{k}$ respectively. Find the acute angle between PQ and the line;
$1 - x = \frac{y-3}{2} = \frac{4-x}{2}$ (5marks)

8. Solve the differential equation, $\left(\frac{dy}{dx}\right)^3 = e^{(x-3y)}$. Given that y (6) = 0. (5marks)

SECTION B (60MARKS)
Attempt ONLY 5 questions in this section.

9. a) Show that; $\log_{16}(xy) = \frac{1}{2}\log_4 x + \frac{1}{2}\log_4 y$. Hence or otherwise, solve the simultaneous equations.

$\log_{16}(xy) = \frac{7}{2}$

$\log_4 x / \log_4 y = -8$ (7marks)

b) Solve the equation $2^{(2+2x)} + 3.2^x - 1 = 0$. (5marks)

10. a) Find x, if $\sin^{-1}x + cos^{-1}\left(\frac{x}{2}\right) = \frac{5\bar{\wedge}}{6}$. (5marks)

b) Express $5sin\emptyset + 12cos\emptyset$ in the form $r\ sin(\emptyset + a)$, giving the value of r and a, hence find $5sin\emptyset + 12cos\emptyset = 7$. (7marrks)

11. a) Differentiate with respect to x.

i) $x^{\log_{10} x}$

ii) $tan^{-1}\left(\frac{1-x}{1+x}\right)$, simplify your answers (8marks)

b) if $y = e^{4x}cos3x$, show that $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 25y = 0$. (4marks)

12. a) Show that the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$ and the plane $\underline{r}.\begin{pmatrix}4\\-1\\-3\end{pmatrix} = 4$ are parallel and find the perpendicular distance of the line from the plane. (6marks)

b) Find the equation of the plane passing through the origin and parallel to the lines'

$\frac{x+2}{3} = \frac{y-1}{4} = \frac{z+1}{5}$ and $\frac{x-3}{4} = \frac{y-2}{-5} = \frac{z+1}{1}$. (6marks)

13. a) Solve the differential equation

$x^2\frac{dy}{dx} = y(y + x)$; Given that $y(4) = 6$. (4marks)

b) A certain game park was found to have 100 lions. Given that the lions die at a rate proportional to the number of lions present and the initial death rate is 5 lions per year.

i) Form a differential equation and solve it.

ii) How many lions will be in the park after six years? (8marks)

14. a) Given that $Z = \cos\emptyset + isin\emptyset$, where $\emptyset \neq \pi$,

show that $\frac{2}{1+z} = 1 - i\,tan\left(\frac{1}{2}\emptyset\right)$. (6marks)

b) The polynomial $P(z) = z^4 - 3z^3 + 7z^2 + 21z - 26$ has $2 + 3i$ as one of the roots. Find the other three roots of the equation $P(z) = 0$. (6marks)

15. a) Work out $\int \frac{dx}{e^x - 1}$. (5marks)

b) The area bounded by the curve $y = x(x - 4)$, and the x-axis is rotated about the x-axis through a ½ -turn. Find the volume of the solid generated. (7marks)

16. a) find an equation of the circle that passes through the points.

$A(-1,4), B(2,5)\ and\ C(0,1)$. (5marks)

b) The line $x + y = c$ is a tangent to the circle $x^2 + y^2 - 4y + 2 = 0$. Find the coordinates of the points of contact of the tangent for each value of C. (7marks)

END