## ALTERNATING CURRENT (AC)

This is a generalized form of current or voltage which is periodic. When a source's EMF has its polarity constantly changing with time, it is said to be alternating e,g the output form of an a.c generator. When such a source with alternating polarity is connected to an external complete circuit, it makes the current flowing through the circuit to keep on changing direction with time and is thus known as an alternating current.
Defining:
An alternating current or voltage is one that varies periodically with, both in magnitude and direction.
Examples include:
(a) sinusoidal a.c voltage or current.

(b)square wave a.c

(c)saw-tooth out put


Sinusoidal A.C output.
An alternating current can be considered as sinusoidal when its wave profile variations with time can be represented by a sine wave.


A sinusoidal a.c current is represented by the equation $I=I_{0} \sin \omega t$ and a.c voltage by $V=V_{0} \sin \omega t$, where $\omega=2 \pi f$ and $f$ is the frequency of the a.c source expressed in Hertz (Hz).
Basic definitions:
(i) Peak value of a.c, is the maximum value of the a.c output ( $I_{\circ}$ or $V_{o}$ ).
(ii) Cycle, is one complete alternation of the wave form or profile.
(iii) Wave length $\lambda$, is a distance between two successive crests or troughs of the wave profile.
(iv) Period T , is the time taken to complete one alternation or cycle.
(v) Frequency $f$, is the number of complete cycles or alternations made by the wave profile in one second.
(vi) Root Mean Square value of a.c (r.m.s), is the steady/ direct current that dissipates heat in a given resistor at the same rate as the alternating current.
NB. R.m.s value of current represents the square root of the average values of the squares of current.

## Relationship between $I_{r m s}$ and peak value $I_{0}$

Suppose $I=I_{0} \sin \omega t$ is the instantaneous value of a.c passing through a resistor of resistance $R$ at any time $t 5$, and it produces some heating effect in the resistor.
The average a.c power over one cycle $\langle P\rangle$ is given by;

$$
\begin{align*}
&\langle P\rangle=\left\langle\left.\right|^{2} R\right\rangle_{T} \\
&=\left\langle I_{0}^{2} R \sin ^{2} \omega t\right\rangle_{T} \\
&=I_{0}^{2} R\left\langle\sin ^{2} \omega t\right\rangle_{\top} \\
& \text { But } \sin ^{2} \omega t=\frac{1}{2}(1-\cos 2 \omega t) \\
& \therefore \quad\left\langle\sin ^{2} \omega t\right\rangle=\frac{1}{2} \text { since }\langle\cos 2 \omega t\rangle=0 \\
& \text { Thus }\langle P\rangle=\frac{1}{2} I_{0}{ }^{2} \mathrm{R} \ldots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~ \tag{i}
\end{align*}
$$

But from definition, if direct current $\mathrm{I}_{\mathrm{rms}}$ passes through the same resistor and produces the same power output,, then;

$$
\left.\begin{array}{rl}
\mathrm{P}_{\mathrm{d}} & =\mathrm{I}_{\mathrm{rms}}{ }^{2} \mathrm{R} \ldots \ldots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .(i i) ~
\end{array}\right)
$$

## A.C CIRCUITS

(1) A.C circuits through a resistor of resistance $R$.

Suppose a sinusoidal alternating voltage $\mathrm{V}=\mathrm{Vosin} 2 \pi \mathrm{ft}$ is applied across a resistor of resistance $R$, an instantaneous current is set up to flow in the circuit.


Instantaneous current $I=\frac{V}{R}=\frac{V_{0} \sin \omega t}{R}$

$$
\begin{aligned}
& \text { But } \frac{V_{0}}{R}=I_{0} \\
& \text { Thus } I=I_{0} \sin \omega t=I_{0} \sin 2 \pi f t \\
& \text { And } \mathrm{R}=\frac{\mathrm{V}_{0}}{\mathrm{I}_{0}}=\frac{\mathrm{V}_{\mathrm{rms}} \sqrt{2}}{\mathrm{I}_{\mathrm{rms}} \sqrt{2}}=\frac{\mathrm{V}_{\mathrm{rms}}}{\mathrm{I}_{\mathrm{rms}}}
\end{aligned}
$$

Graphs of V and I on the same axis against time or angle $(\omega t)$


Phasor (vector) diagram
This is a rotating vector in the anticlockwise direction representing the stationary values of voltage and current and the time lag (phase) between their peak values ( $\mathrm{V}_{\mathrm{o}}$ and $I_{0}$ ) or $V_{\text {rms }}$ and $I_{r m s}$
For a resistor, the phasor diagram appears as shown below.


From the equation $R=\frac{V_{0}}{I_{0}}=\frac{V_{\text {rms }}}{I_{\text {rms }}}$ it shows that $R$ is independent of
frequency. i.e


Power dissipated or absorbed in a resistor.

$$
\text { From } \begin{aligned}
\mathrm{V} & =\mathrm{V}_{0} \sin \omega t \text { and } \mathrm{I}=\mathrm{I}_{0} \sin \omega t \\
\mathrm{P} & =\mathrm{VI}=\mathrm{V}_{0} \sin \omega t \mathrm{I}_{0} \sin \omega t \\
\mathrm{P} & =\mathrm{I}_{0} \mathrm{~V}_{0} \sin ^{2} \omega t
\end{aligned}
$$

Average power absorbed in a resistor over one complete cycle is given by;

$$
\begin{aligned}
\langle P\rangle_{\mathrm{T}} & \left.=<\mathrm{I}_{\mathrm{O}} \mathrm{~V}_{\mathrm{o}} \sin ^{2} \omega \mathrm{t}\right\rangle \\
& \left.=\mathrm{I}_{\mathrm{O}} \mathrm{~V}_{\mathrm{o}}<\sin ^{2} \omega \mathrm{t}\right\rangle \\
& \left.=I_{0} \mathrm{~V}_{\mathrm{O}}<\frac{1}{2}(1-\cos 2 \omega \mathrm{t})\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
\text { But }\left\langle\frac{1}{2}(1-\cos 2 \omega t)\right\rangle_{T} & =\frac{1}{2} \\
\therefore\langle P\rangle_{T} & =\frac{1}{2} I_{0} V_{0} \text { or }\langle P\rangle_{T}=I_{0}^{2} R \text { or }\langle P\rangle_{T} I_{r m s} V_{r m s}
\end{aligned}
$$

(2) A.C through capacitors

A pure capacitor is a capacitor having the space between the plates filled with a dielectric material of infinite resistance to current flow.
Suppose a sinusoidal alternating voltage $\mathrm{V}=\mathrm{V}_{0} \sin \omega \mathrm{t}$ or $\mathrm{V}=\mathrm{V}_{0} \sin 2 \pi f t$ is applied across the plates of the capacitor of capacitance $C$;


As the voltage goes on increasing, charge is deposited on each of the plates of magnitude Q when the p .d across the plates is maximum.

$$
\begin{aligned}
& \mathrm{Q}=\mathrm{CV} \\
& \mathrm{Q}=\mathrm{C} \mathrm{~V}_{0} \sin \omega \mathrm{t} .
\end{aligned}
$$

The instantaneous current I flowing in the circuit at any time $t$ is given by;

$$
\begin{aligned}
\mathrm{I} & =\frac{d Q}{d t}=\frac{d}{d t}\left(\mathrm{CV}_{0} \sin \omega \mathrm{t}\right) \\
\mathrm{I} & =\mathrm{CV}_{0} \frac{d}{d t}(\sin \omega \mathrm{t}) \\
\mathrm{I} & =\mathrm{CV}_{0} \omega \cos \omega \mathrm{t} \\
\text { But } \quad \mathrm{CV} & =\mathrm{I}_{0}(\text { peak value }) \\
\therefore \quad \mathrm{I} & =\mathrm{I}_{0} \cos \omega \mathrm{t} \text { or } \mathrm{I}=\mathrm{I}_{0} \cos 2 \pi \mathrm{ft}
\end{aligned}
$$

$$
\begin{aligned}
& \text { From } \mathrm{CV}_{0} \omega=\mathrm{I}_{0} ; \frac{\mathrm{V}_{0}}{\mathrm{I}_{0}}=\frac{1}{\mathrm{C} \omega}=\mathrm{X}_{\mathrm{c}} \\
& \text { Thus } \mathrm{X}_{\mathrm{c}}=\frac{1}{C \omega}=\frac{1}{2 \pi f C}
\end{aligned}
$$

Where $X_{c}$ is the reactance of a capacitor (capacitive reactance) and its S.I unit is an ohm ( $\Omega$ )

## Definition:

Capacitive reactance $X_{c}$ is the non-resistive opposition offered by a capacitor to the passage of changing current through the capacitor.
From $\mathrm{X}_{\mathrm{c}}=\frac{1}{C \omega}=\frac{1}{2 \pi f C}$ where C is a constant, $\mathrm{R} \propto \frac{1}{f}$
A graph of $X_{c}$ against frequency appears as shown below.


From $V=V_{o} \sin \omega t$.
$\mathrm{I}=\mathrm{I}_{\mathrm{o}} \cos \omega \mathrm{t}=\mathrm{I}_{\mathrm{o}} \sin (\omega \mathrm{t}+90)$
Instantaneous current I through a capacitor leads voltage $V$ by $90^{\circ}$ or $\frac{\pi}{2}$ radians. Thus the phase difference $\Phi=90^{\circ}$ or $\Phi=\frac{\pi}{2}$ rad between I and V .
Graphs of I and V on the same axis plotted against time or angle have the shapes shown below.


Phasor diagram.


Power absorbed in a capacitor.
From $V=V_{0} \sin \omega t$ and $I=I_{0} \cos \omega \mathrm{t}$,
Instantaneous power $\mathrm{P}=\mathrm{IV}=\mathrm{I}_{\mathrm{o}} \mathrm{V}_{\mathrm{o}}(\sin \omega \mathrm{t} \cos \omega \mathrm{t})$

$$
\begin{aligned}
& \mathrm{P}=\frac{1}{2} I_{0} \mathrm{~V}_{0}(2 \sin \omega \mathrm{t} \cos \omega \mathrm{t}) \\
& \mathrm{P}=\frac{1}{2} I_{0} \mathrm{~V}_{0}(\sin 2 \omega \mathrm{t})
\end{aligned}
$$

Average power absorbed in a capacitor over a complete cycle is given by;

$$
\begin{aligned}
& \quad\langle P\rangle_{T}=\frac{1}{2} I_{o} V_{o}\langle(\sin 2 \omega t)\rangle_{T} \text {, but }\langle(\sin 2 \omega t)\rangle_{T}=0 \\
& \text { Thus }\langle P\rangle_{T}=0
\end{aligned}
$$

Hence no power is absorbed in a capacitor and thus often known as a wattles component.

## Explanation.

During the first quarter of the a.c input, the capacitor charges up to a maximum value equal to the p.d of the source. During the second quarter, when the current through the circuit decreases from maximum to zero, the p.d across the capacitor then begins to drop up to zero volts. The energy originally stored in the electric field of the capacitor is given back to the source.
During the third quarter of the cycle when the current changes direction, the capacitor charges again in the opposite sense to the first and energy is stored in the electric field of the capacitor. During the last quarter of the cycle, the p.d across the capacitor plates rapidly drops to zero due to decreasing current and all the energy stored in the capacitor is given back to the source as the capacitor discharges. Therefore, in one complete cycle, the capacitor charges twice and discharges twice, thus remaining with no energy stored in its electric field between the plates hence power absorbed is zero.
(3)A.C through an inductor.

An inductor is a coil wound on an insulator. A pure inductor is one with negligible resistance to the passage of current through it. An inductive coli is a coil that opposes the passage of changing current through it.
Suppose a changing current $\mathrm{I}=\mathrm{I}_{0} \sin \omega \mathrm{t}$ is passed through an inductor of self inductance $L$ when a p.d is applied across it.


When this changing current passes through the inductor, a back emf $\mathrm{E}_{\mathrm{b}}$ is generated.

$$
\begin{aligned}
\text { But } \mathrm{E}_{\mathrm{b}} & =-\mathrm{L} \frac{d I}{d t}=-\mathrm{L} \frac{d}{d t}\left(\mathrm{I}_{0} \sin \omega \mathrm{t}\right) \\
\mathrm{E}_{\mathrm{b}} & =-\mathrm{L} \mathrm{I}_{0} \omega \cos \omega \mathrm{t}
\end{aligned}
$$

However, back emf Eb opposes the applied voltage V , thus $\mathrm{E}_{\mathrm{b}}=-\mathrm{V}$

$$
-V=-L I_{0} \omega \cos \omega t
$$

$$
\therefore \mathrm{V}=\mathrm{L} \mathrm{I}_{0} \omega \cos \omega \mathrm{t}
$$

Thus $\mathrm{V}=\mathrm{V}_{0} \cos \omega \mathrm{t}$, where $\mathrm{V}_{0}=\mathrm{LI}_{0} \omega$, the peak voltage across the inductor.

$$
\text { From } \mathrm{V}_{0}=\mathrm{LI}_{0} \omega, \quad \frac{\mathrm{~V}_{0}}{\mathrm{I}_{0}}=\mathrm{L} \omega=\mathrm{X}_{\mathrm{l}}
$$

Thus $X_{L}=\omega L=2 \pi f$ hence $X_{I} \alpha f$
A graph of reactance XI of an inductor against frequency f is as shown below.


The S.I unit of $X_{L}$ is an ohm ( $\Omega$ )
From $\mathrm{I}=\mathrm{I}_{0} \sin \omega \mathrm{t}$ and $\mathrm{V}=\mathrm{V}_{\mathrm{o}} \cos \omega \mathrm{t}=\mathrm{V}_{\mathrm{o}} \sin \left(\omega \mathrm{t}+\frac{\pi}{2}\right)$, voltage V leads current I by $\frac{\pi}{2}$ radians or $90^{\circ}$
Graphs of I ad V through an inductor against time appear as shown below.


Phasor diagram


Power absorbed in an inductor.

$$
\text { From } V=V_{o} \cos \omega t \text { and } I=I_{0} \sin \omega t \text {, }
$$

Instantaneous power $\mathrm{P}=\mathrm{IV}=\mathrm{I}_{\mathrm{o}} \mathrm{V}_{\mathrm{o}}(\sin \omega \mathrm{t} \cos \omega \mathrm{t})$

$$
\begin{aligned}
& \mathrm{P}=\frac{1}{2} I_{0} \mathrm{~V}_{0}(2 \sin \omega \mathrm{t} \cos \omega \mathrm{t}) \\
& \mathrm{P}=\frac{1}{2} I_{0} \mathrm{~V}_{0}(\sin 2 \omega \mathrm{t})
\end{aligned}
$$

Average power absorbed in an inductor over a complete cycle is given by;

$$
\begin{aligned}
\langle P\rangle_{T} & \left.=\frac{1}{2} I_{0} V_{0}<(\sin 2 \omega t)\right\rangle_{T}, \text { but }\langle(\sin 2 \omega t)\rangle_{T}=0 \\
\text { Thus }\langle P\rangle_{T} & =0
\end{aligned}
$$

Hence no power is expended in a pure inductor.
Explanation.
When a.c is passed through the inductor, during the $1^{\text {st }}$ quarter of the cycle, current is increasing and produces an increasing magnetic field through the inductor thus an emf called the back emf is induced in the coil and energy is stored in the magnetic field of the inductor. During the $2^{\text {nd }}$ quarter of the cycle, current begins to reduce in the circuit and the magnetic flux linking the coil also reduces. The energy originally stored in the inductor is given back to the source in an attempt to maintain the decaying current through the inductor.
During the $3^{\text {rd }}$ quarter of the cycle, current increases and flows through the inductor in the opposed direction and thus an increasing magnetic field build up in which energy is stored. In the $4^{\text {th }}$ quarter of the a.c cycle, current drops and the energy originally stored in the magnetic field is given back to the source to enhance the decaying current. Hence in one complete cycle of an a.c through an inductor, no energy is retained in it.

## L. R. C Circuits

Summary of phase relations for a resistor, capacitor and an inductor.
Resistor


Capacitor


Inductor


L R series circuit


$$
V^{2}=(I R)^{2}+\left(I X_{L}\right)^{2}
$$

$$
V=I \sqrt{\left(R^{2}+X_{L}{ }^{2}\right)}
$$

$$
\frac{V}{I}=\sqrt{\left.\left(\mathrm{R}^{2}+\mathrm{X}_{\mathrm{L}}{ }^{2}\right)=\sqrt{\left(\mathrm{R}^{2}+(2 \pi \mathrm{f})^{2}\right)}, \overrightarrow{2}\right)}
$$

But $\quad \frac{V}{I}=\mathrm{Z}$, called impedance.
Thus $Z=\sqrt{\left(R^{2}+(2 \pi f)^{2}\right)}$.

$$
\tan \Phi=\frac{\mathrm{V}_{\mathrm{L}}}{\mathrm{~V}_{\mathrm{R}}}=\frac{\perp \mathrm{X}_{\mathrm{L}}}{\mathbb{R}}
$$

Phase angle $\Phi \tan ^{-1}\left[\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}\right]$
L C series circuit.


I. R.C Series circuit.


$$
\begin{aligned}
& V^{2}=\left(V_{L}-V_{C}\right)^{2}+V_{R}{ }^{2} \\
& V=\sqrt{ }\left(\left(V_{L}-V_{C}\right)^{2}+V_{R}{ }^{2}\right) \\
& V=\sqrt{ }\left(\left(I X_{L}-I X_{C}\right)^{2}+(I R)^{2}\right) \\
& V=I \sqrt{ }\left(\left(X_{L}-X_{C}\right)^{2}+R^{2}\right)
\end{aligned}
$$



Impedance of L.R.C circuits, $Z=\frac{V_{\text {rms }}}{I_{\text {rms }}}=\sqrt{\left(\left(X_{L}-X_{C}\right)^{2}+R^{2}\right), ~(1) ~}$ $\qquad$

$$
\tan \Phi=\frac{\mathrm{V}_{\mathrm{L}}-\mathrm{V}_{\mathrm{C}}}{\mathrm{~V}_{\mathrm{R}}}=\frac{\mathrm{I}\left(\mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{C}}\right)}{\mathrm{IR}}
$$

$$
\Phi=\tan ^{-1}\left[\frac{\mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{C}}}{\mathrm{R}}\right]
$$

From equation (1) above when the value of $C$ is adjusted until $X_{C}=X_{L}$, resonance is said to have occurred at a frequency $f_{o}$ and $Z=R, V_{L}=V_{C}$

$$
\begin{aligned}
& \text { When } \quad \begin{aligned}
X_{C} & =X_{L} \\
\frac{1}{2 \pi f_{0} C} & =2 \pi f_{0} L \\
f_{o} & =\frac{1}{2 \pi \sqrt{L C}}
\end{aligned}
\end{aligned}
$$

A graph showing $Z, R, X_{L}$ and $X_{C}$ against frequency for a resonant circuit appears as shown below.


## Examples

(1) A sinusoidal alternating voltage of $V=8 \sin 120 \pi t$ volts is connected across of a resistor of $6 \Omega$. Find the mean power dissipated in the resistor. Hence deduce the r.m.s value of the current.

Solution;
From $\mathrm{V}=8 \sin 120 \pi \mathrm{t}, \mathrm{V}_{0}=8 \mathrm{~V}$

$$
\begin{aligned}
& \langle P\rangle=\frac{V_{o}^{2}}{2 R}=\frac{8^{2}}{2 \times 6}=5.33 \mathrm{~W} . \\
& \langle P\rangle=\frac{1}{2} I_{o} V_{o}=5.33, I_{o}=\frac{5.33 \times 2}{8}=1.33 \mathrm{~A} \\
& \text { But } I_{\text {rms }}=0.707 I_{0}, I_{r m s}=0.94 \mathrm{~A} .
\end{aligned}
$$

(2)A sinusoidal voltage of 20 V (rms) and frequency 60 Hz is applied across an inductor of inductance 0.2 H and negligible resistance. Calculate the root mean square value of the current which flows through the coil. And hence write the expression for the instantaneous current through the circuit.

Solution.
$V_{\text {rms }}=20 \mathrm{~V}, \mathrm{f}=60 \mathrm{~Hz}, \mathrm{~L}=0.2 \mathrm{H}$
$V_{o}=\frac{V_{\mathrm{rms}}}{0.707}=\frac{20}{0.707}=28.29 \mathrm{~V}$.
But vo-ょw $l_{0}, I_{0}=\frac{28.29}{2 \pi x 60 \times 0.2}=0.39 \mathrm{~A}$
Also $I_{\text {rms }}=0.707 \mathrm{I}_{\mathrm{o}}, \mathrm{I}_{\mathrm{rms}}=0.707 \times 0.39=0.28 \mathrm{~A}$
Hence $I=0.39 \cos 120 \pi t$
(3) A capacitor of $0.1 \mu$ Fis in series with an a.c source of frequency 500 Hz . If the r.m.s value of the current flowing is 6 mA , calculate the voltage across the capacitor.

## Solution

$\mathrm{C}=0.1 \times 10^{-6} \mathrm{~F}, \mathrm{f}=500 \mathrm{~Hz}, \mathrm{I}_{\mathrm{rms}}=6 \times 10^{-3} \mathrm{~A}$

$$
\begin{aligned}
& \mathrm{X}_{\mathrm{C}}=\frac{1}{2 \pi f C}=\frac{1}{2 \pi \times 500 \times 0.1 \times 10^{-6}}=3183.1 \Omega \\
& \text { but } \mathrm{X}_{\mathrm{C}}=\frac{\mathrm{V}_{\mathrm{rms}}}{\mathrm{I}_{\mathrm{rms}}}, \mathrm{~V}_{\mathrm{rms}}=3183.1 \times 6 \times 10^{-6}=19.1 \mathrm{~V}
\end{aligned}
$$

(4) A bulb is connected in series with an inductive coil and a d.c source as shown below.

(i)What happens to the brightness of the bulb when an iron core is inserted in the coil?
(ii) Explain what happens to the brightness of the bulb when the d.c source is replaced with a.c and an iron core is inserted in the coil.

## Solution

(i) no change in the brightness of the bulb.
(ii) brightness decreases.

When a.c is connected, the magnetic flux in the coil changes. Inserting the iron core enhances the changing magnetic flux linking the coil hence increasing the back emf in the coil. The increase in the back emf reduces the amount of current flowing through the circuit and so decreasing the brightness.
(5)A reactive circuit consists of a coil of self reactance $2.0 \times 10^{-4} \mathrm{H}$ and resistance $10 \Omega$ connected in series with a variable air capacitor and an a.c source of 0.10 V (r.m.s) operating at 1.0 MHz . Calculate;
(i)the capacitance of a capacitor that gives resonance in the circuit.
(ii)the current flowing in the circuit.
(ii)the p.ds across the coil and capacitor at resonance.

Solution
$\mathrm{L}=2.0 \times 10^{-4} \mathrm{H}, \mathrm{R}=10 \Omega, \mathrm{~V}_{\mathrm{rms}}=0.10 \mathrm{~V}, \mathrm{f}_{\mathrm{o}}=10^{6} \mathrm{~Hz}$
(i) At resonanc $\frac{1}{2 \pi f_{0} C} \quad=2 \pi f_{0} L, \quad C=\frac{1}{4 \pi^{2} f_{0}{ }^{2} L}$

$$
C=\frac{1}{4 \pi^{2}\left(10^{6}\right)^{2} \times 2.0 \times 10^{-4}}=1.267 \times 10^{-10} \mathrm{~F}
$$

(ii)

$$
\mathrm{I}=\frac{V}{R}=\frac{0.10}{10}=0.01 \mathrm{~A}
$$

(iii)

$$
\begin{aligned}
& \mathrm{X}_{\mathrm{C}}=\frac{1}{2 \pi f C}=\frac{1}{2 \pi \times 10^{6} \times 1.267 \times 10^{-10}}=1.256 \mathrm{~K} \Omega \\
& \mathrm{~V}_{\mathrm{C}}=I \mathrm{X}_{\mathrm{C}}=0.01 \times 1256=12.56 \mathrm{~V} \\
& \mathrm{VL}=2 \pi \mathrm{fLI}=2 \pi \times 10^{6} \times 2 \times 10^{-4} \times 0.01=12.56 \mathrm{~V}
\end{aligned}
$$

## A.c meters

Unlike moving coil instruments whose direction of deflection of the pointer depends on the direction of current I through the instrument, a.c meters have their pointer deflection being independent of the direction of flow of current. Thus moving coil instruments can not be used for measurement of a.c because they just produce vibrations of the pointer about one position (zero position).

## Types of a.c meters.

(1) The hot wire ammeter.


Mode of operation.
The current to be measured is passed into the instrument via a fine resistance wire $X Y$. This current causes XY to heat up, expand and sags.
The sag is then picked up by another wire PQ wound round a smooth grooved pulley and kept taut by springs fastened to the base of the instrument. As the spring pulls the wire PQ downwards, the pulley turns about its pivot together with the pointer attached to it making the pointer to move over the non-linear scale through an angle $\theta$. The deflection $\theta$ is proportional to the mean or the average of the square current i.e $\left.\theta \alpha<\left.\right|^{2}\right\rangle$ hence the square scale or non linear scale.
(2) The moving iron meter
(a) repulsive type.

mode of operation.
Current I to be measured is fed into the coil (solenoid) via the terminals $T_{1}$ and $T_{2}$ of the coil. The current through the coil causes the solenoid to produce a magnetic field with in it and magnetizes the two soft iron rods $P$ and $Q$ in the same sense (i.e adjacent ends of $P$ and $Q$ have the same polarity) irrespective of the direction of current through the coil. The two iron rods with the same polarity then repel each other with a magnetic force that is proportional to the square of the current through the coil. The soft iron rod $P$ then moves off from rod $Q$, causing the pointer attached to it to turn about the pivot, and thus deflecting over the scale through an angle $\theta$ and it is only stopped by the restoring torque due to the pair of control hair springs. The deflection $\theta$ is proportional to the mean of the square current hence the square scale.
(b) The attractive type of moving iron meter.

## Structure.

It consists of a specially designed piece of soft iron material placed just at the ends along the axis of the open end of the solenoid.
A pointer is attached to the soft iron piece with its tip along a non-linear scale. The deflection of the pointer is controlled by a pair of hair springs along the axis of the soft iron piece and an air dash pot to control the damping of the system.


When current I to be measured is fed into and out of the solenoid via terminals $\mathrm{T}_{1}$ and $T_{2}$, a magnetic field is created inside the solenoid and links the specially designed soft iron piece of material. Irrespective of the direction of current flow through the solenoid, the pole of the coil at the open end attracts the tip of the soft iron piece towards it with a magnetic force proportional to the square of current I. This then causes the pointer attached to the soft iron piece to turn together with the iron piece about the pivot through an angle $\theta$, over the scale until stopped by the restoring torque due to a pair of control hair springs.
(3)The thermo couple meter

It consists of an evacuated bulb having a fine resistance wire XY passing through its center and carrying a current I. Attached to the center of XY inside the evacuated bulb are two wires of different materials P and Q having their ends at room temperature and connected to a micrometer $(\mu \mathrm{A})$ calibrated to measure $I_{\text {rms }}$ of current I .


Current to be measured is passed through the fine resistance wire XY which then gets heated up.point $R$ of the wire $X Y$ acts as the hot junction and is being shielded off from droughts by an evacuated bulb. The wires $P$ and $Q$ of different materials are connected to point $R$ and their cold junctions $A$ and $B$ respectively are at room temperature. The temperature gradient between the two junctions produces a thermoelectric emf across the junctions and thus current flows through the meter $(\mu \mathrm{A})$ which is calibrated to measure the $I_{\text {rms }}$ of the a.c fed into XY . The $\mathrm{I}_{\text {rms }}$ produces a deflection $\Phi$ on the scale, where $\Phi \propto \mathrm{I}_{\text {rms }}$

## Rectification.

An alternating current can be converted into direct current by making use of a device called a rectifier e.g thermionic diode, semi-conductor diode, or metal rectifier. The rectifier is connected to the source so that it conducts when it is forward biased. However, if it is reverse biased, then it does not conduct. The figures below show the above conditions.


The graph below shows current-voltage curve of a rectifier.


Have wave rectification.


The rectifier conducts only during half of the cycle which makes A positive. Although the output alternately increases and decreases it is unidirectional.


Full wave rectification.


Full wave rectification is achieved by using an arrangement of four rectifiers known as bridge rectifier. When $N$ is positive, $B$ and $D$ conduct; when $M$ is positive, $C$ and $A$ conduct. In each case the current through the load is in the same direction from $P$ to Q. the p.d across the load has the form below.


In this rectification, the load draws current from the supply on each half of the cycle and so the power utilized is double that achieved with half wave rectification.
The output p.d produced in both rectifiers can be made steadier by putting a suitable capacitor in parallel with the load.


Variation of p.d across load for half wave rectification.
p.d across
smoothed p.d
Variation of $p . d$ across the load for full wave rectification.
p.d across


At points such as A the p.d across the load is maximum and the capacitor is fully charged. In the absence of a capacitor, the p.d would start to fall to zero along the dotted lines. However, immediately the p.d across the load starts to fall, it becomes less than that across the capacitor and thus the capacitor starts to discharge and drives current through the load in the same direction as it flowed during charging. If the time constant of the capacitor-load combination is large, the p.d across the load falls by only a small amount before it starts to rise.

## Exercise

(1)(a)


A coil of inductance $L$ is connected to a source of alternating current as shown above. If the current in the coil is given by $I=I_{0} \sin \omega t$,
(i)Find the expression for the voltage V across the coil.
(ii) Sketch using the same axes, graphs to show the variation of $V$ and I with time and comment on the graphs.
(b)Explain why a capacitor allows the flow of alternating current but not direct current.
(c)(i) With the aid of a labeled diagram, explain how a repulsive type of moving iron meter works.
(ii)state two advantages of a moving iron meter over the moving coil.
(d) A heater of resistance $500 \Omega$ consumes energy equivalent to 15 KWh when connected to an a.c source in 10 hours. Calculate the peak value of the source. (2)(a) (i) The instantaneous value of a sinusoidal voltage is give by $\mathrm{V}=\mathrm{Vosin} 60 \pi t$ volts. Calculate the r.m.s value and the frequency of the alternating voltage.
(ii)Show that $I_{\text {rms }}=0.707 I_{\text {。 }}$
(b) With the aid of a labeled diagram, describe how a hot iron meter works.
(c) An alternating current I flows through the coil of inductance L. the instantaneous value of current is $I=200 \sin 4 \pi f t$.
(i) Derive the expression for the voltage V across the coil.
(ii)State the phase of $V$ relative to that of $I$
(iii) Show that a pure inductor is "wattles" component in an a.c circuit.
(d) A $1000 \mu \mathrm{~F}$ capacitor is joined with a $2.5 \mathrm{~V}, 0.3 \mathrm{~A}$ lamp and a 500 Hz supply.

## Calculate;

(i) root mean square voltage of supply to the lamp
(ii) The p.d across the capacitor.
(iii) The phase angle between current and voltage of the capacitor.

In the figure below, a resistor R and bulb $\mathrm{B}_{1}$ are in series and are connected in parallel with an air-cored inductor $L$ which is in series with bulb $\mathrm{B}_{2}$.

(i)explain what happens when switch K is closed.
(ii)explain what happens when the air-cored inductor is replaced with an iron cored inductor.
(3) (a) Define;
(i) root mean square value of an alternating voltage.
(ii) reactance of a capacitor
(b) A capacitor of capacitance C is connected across a source of alternating voltage $V=V_{0} \sin \omega t$.
(i) find the current which flows in the circuit.
(ii)sketch using the same axes, the voltage across the capacitor and the current which flows in the circuit, with time.
(iii) explain the phase difference between the voltage and current in (ii) above.
(4)(a) Differentiate between root mean square value and peak value of an alternating current.
(b)(i)An alternating voltage is applied across a capacitor of capacitance, C. show that the current in the circuit leads the voltage by $\frac{\pi}{2}$.
(ii) Find the expression of the capacitive reactance in terms of frequency, $f$ and capacitance, C .

